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On rank 3 residually connected geometries for M_{23}

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Abstract. In this paper we determine all rank 3 residually connected geometries for the Mathieu group M_{23} for which object stabilizers are maximal subgroups.

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1. Introduction and Notation

We begin by reviewing geometries and some standard notation. Let I, Γ be sets, where I is a finite and let t be map from Γ to I. Then the triple $(\Gamma, *, t)$, where $*$ is a symmetric incidence relation on Γ, is a geometry provided that whenever $x * y$ $(x, y \in \Gamma)$ then $t(x) \neq t(y)$. It is usual we will do it here, to write Γ instead of $(\Gamma, *, t)$ and to say Γ is a geometry. The map $t : \Gamma \rightarrow I$ is called the type map and we say $x \in \Gamma$ has type i if $t(x) = i$. Also for $x, y \in \Gamma$ if $x * y$, then we will say x and y are incident. The rank of the geometry is the cardinality of $t(\Gamma)$. For $i \in I$, $\Gamma_i = \{x \in \Gamma \mid t(x) = i\}$; so Γ_i consist of all elements of Γ which have type i. Suppose Γ is a geometry, for $x \in \Gamma$, the residue of x is $\Gamma_x = \{y \in \Gamma \mid x*y\}$. The notation of the residue is important in the theory of geometries note that $(\Gamma_x, * \mid \Gamma_x, t)$ is a geometry in its own right (where $* \mid \Gamma_x$ is the restriction of $*$ to Γ_x). Also we note that for every $y \in \Gamma_x$, $t(x) \neq t(y)$. A flag F of Γ is a subset of Γ which, for all $x, y \in F$, $x \neq y$, $x * y$. Let Γ be a geometry and F a flag of Γ. The type of F is the subset $t(F)$ of I and the rank (respectively corank) of F is the cardinality of $t(F)$ (respectively $I \setminus t(F)$). A chamber of Γ is flag of rank I. All geometries we consider are assumed to contain at least one flag of rank $\mid I \mid$. The automorphism group of Γ, $Aut\Gamma$, consist of all permutations of Γ which preserve the sets Γ_i and the incidence relation $*$. Let G be a subgroup of $Aut\Gamma$. We call Γ a flag transitive geometry for G if for any two flags F_1 and F_2 of Γ having the same type, there exists $g \in G$ such that ${F_1}^g = F_2$.

A geometry Γ is called residually connected if for all flags F of Γ of corank 2 the incidence graph of Γ_F is connected. Now suppose that Γ is a flag transitive geometry for the group G. As is well-known we may view Γ in terms of certain cosets of G. This is the approach we shall follow here. For each $i \in I$ choose an $x_i \in \Gamma_i$ and set $G_i = Stab_G(x_i)$. Let $\mathcal{F}$=$\{G_i : i \in I\}$. We now define a geometry $\Gamma(G, \mathcal{F})$ where the objects of type i in $\Gamma(G, \mathcal{F})$ are the right cosets of G_i in G and for $G_i x$ and $G_j y$ $(x, y \in G, i, j \in I)$ $G_i x \star G_j y$ whenever $G_i x \cap G_j y \neq \emptyset$. Also by letting G act upon $\Gamma(G, \mathcal{F})$ by right multiplication we see that $\Gamma(G, \mathcal{F})$ is a flag transitive geometry for G. Moreover Γ and $\Gamma(G, \mathcal{F})$ are isomorphic geometries for G. So we shall be studying geometries of the form $\Gamma(G, \mathcal{F})$, where $G \cong M_{23}$ and G_i is a maximal subgroup of G for all $i \in I$.

A numerical summary of our results is given in

Theorem 1. *Suppose $G \cong M_{23}$ and Γ is residually connected flag transitive geometry for G with G_i is a maximal subgroup of G. If Γ has rank 3, then up to conjugacy in $AutG$, then there are 1050 possibilities for Γ.*

In Section 2, we give explicit descriptions of these geometries making heavy use of the degree 23 permutation representation for M_{23}. We use GAP for some part of our calculation see [6]; especially calculation for 23 : 11.

Beukenhout, in [1], sought to give a wider view of geometries so as to encompass configurations observed in the finite sporadic simple groups. An outgrowth of this has been attemps to catalogue various subcollections of geometries for the finite sporadic simple groups (and other related groups). So - called minimal parabolic geometries and maximal 2-local geometries were investigated in [17] and [18] while geometries satisfying certain additional conditions for a number of (relatively) small order simple groups have been exhaustively examined. See, for example, [2], [5],[11], [12], [13], [14], [15], [16] and [19]. the results in most of these papers were obtained using various computer algebra systems. All rank2 and rank 3 residually connected geometries of M_{22} (the Mathieu Group of degree 22) were investigated in [8]. Kilic, in [9], calculated all rank 2 geometries for the Mathieu group M_{24}. Again Kilic, in [10], calculated all rank 2 geometries for the Mathieu group M_{23} for which object stabilizers are maximal subgroups. Now we determine all rank 3 residually connected flag transitive geometries for the Mathieu group M_{23} (the Mathieu Group of degree 23) whose object stabilizer are maximal subgroups.

In this paper we shall use the result of all rank 2 geometries of M_{23} calculated in [10].

For the remainder of this paper G will denote M_{23}, the Mathieu Group of degree 23. Also Ω will denote a 24 element set possessing the Steiner system $S(24, 8, 5)$ as described by Curtis's MOG [4]. We will follow the notation of [4].

So $\Omega = \begin{array}{|c|c|c|}\hline O_1 & O_2 & O_3 \\ \hline \end{array} = \begin{array}{|cc|cc|cc|}\hline \infty & 14 & 17 & 11 & 22 & 19 \\ 23 & 8 & 4 & 13 & 1 & 9 \\ 3 & 20 & 16 & 7 & 12 & 5 \\ 15 & 18 & 10 & 2 & 21 & 6 \\ \hline \end{array}$, where O_1, O_2 and O_3 are the heavy bricks of the MOG. Here M_{24} is the Mathieu group of degree 24 which leaves invariant the Steiner system $S(24, 8, 5)$ on Ω. Set $\Lambda = \Omega \setminus \{\infty\}$

An octad of Ω is just an 8-element block of the Steiner system and a subset of Ω is called a dodecad if it is the symmetric difference of two octads of Ω which intersect in a set of size two. Corresponding to each 4 points of Ω there is a partition of the 24 points into 6 tetrads with the property that the union of any two tetrads is an octad, this configuration will be called a sextet. The following sets will appear when we describe geometries for G.

(i) $\mathcal{D}$=$\{X \subseteq \Lambda||X| = 2\}$ (duads of Λ).
(ii) $\mathcal{H}$=$\{X \subseteq \Lambda|X\cup\{\infty\}$ is an octad of Ω $\}$(heptads of Λ).
(iii) $\mathcal{O}$=$\{X \subseteq \Lambda|X$ is an octad of $\Omega\}$ (octads of Λ).
(iv) $\mathcal{D}_{\rm o}$=$\{X \subseteq \Lambda|X$ is a dodecad of $\Omega\}$ (dodecads of Λ).
(v) $\mathcal{S}$=$\{X_i \subseteq \Omega||X_i| = 4$ (for each $i \in I$), $X_i \cup X_j$ is an octad $(i \neq j)$ and $\Omega = \dot{\bigcup\limits_{i\in I}} X_i$, $i \in I = \{1...6\}\}$ (sextets of Ω).

From the [3], the conjugacy classes of the maximal subgroups of G are as follows:

Order	*Index*	M_i	*Description*
443520	23	$M_1 \cong M_{22}$	$M_1 = Stab_G\{a\}$, $a \in \Lambda$
40320	253	$M_2 \cong L_3(4) : 2b$	$M_2 = Stab_G\{X\}$, $X \in \mathcal{D}$
40320	253	$M_3 \cong 2^4 : A_7$	$M_3 = Stab_G\{X\}$, $X \in \mathcal{H}$
20160	506	$M_4 \cong A_8$	$M_4 = Stab_G\{X\}$, $X \in \mathcal{O}$
7920	1288	$M_5 \cong M_{11}$	$M_5 = Stab_G\{X\}$, $X \in \mathcal{D}_{\rm o}$
5760	1771	$M_6 \cong 2^4 : (3 \times A_5) : 2$	$M_6 = Stab_G\{X\}$, $X \in \mathcal{S}$
253	40320	$M_7 \cong 23 : 11$	

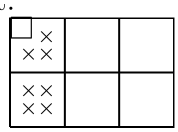
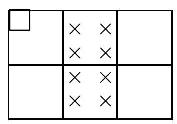
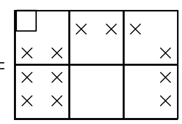
For $i \in \{1, ..., 7\}$, we let $\mathfrak{M}_{\rm i}$ denote the conjugacy class of M_i, M_i as given in the previous table. We also set $\mathfrak{M}$=$\bigcup_{i=1}^{7} \mathfrak{M}_{\rm i}$; so $\mathfrak{M}$ consist of all maximal subgroups of G. In [4] and [9], we can find further information about 23:11. Also put $\mathfrak{X} = \Lambda \cup \mathcal{D} \cup \mathcal{H} \cup \mathcal{O} \cup \mathcal{D}_{\rm o} \cup \mathcal{S}$.

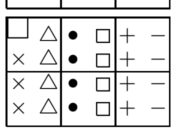
Suppose G_1 and G_2 are maximal subgroups of G with $G_1 \neq G_2$. Set $G_{12} = G_1 \cap G_2$. We use $\mathfrak{M}_{\rm ij}(t)$ to describe $\{G_1, G_2, G_1 \cap G_2\}$ according to the following scheme: $G_1 \in \mathfrak{M}_{\rm i}$, $G_2 \in \mathfrak{M}_{\rm j}$ (and so $G_1 = Stab_G(X_1)$ and $G_2 = Stab_G(X_2)$ for some appropriate subsets X_1 and X_2 of Λ in $\mathfrak{X}$) with $|X_1 \cap X_2| = t$. When listing up the rank 2 geometries of G in [10] the notation $\mathfrak{M}_{\rm ij}(t)$ is not sufficient enough to describe the geometries up to conjugacy in $AutG$. All calculations in $2^4 : (3 \times A_5) : 2$ and $23 : 11$ we can not use this notation, we shall use the following notation; $\mathfrak{M}_{46}(1)$ means the first case of the intersection of octad and sextet, $\mathfrak{M}_{46}(2)$ means the second case of the intersection of octad and sextet. In [10], we shall find more information about it.

Now suppose we have three distinct maximal subgroups of G; G_1, G_2 and G_3. We shall use use G_{12},G_{13}, G_{23} and G_{123} to denote, respectively $G_1 \cap G_2$, $G_1 \cap G_3$, $G_2 \cap G_3$ and $G_1 \cap G_2 \cap G_3$. We extend the above notation using $\mathfrak{M}_{ijk}(t_{ij}, t_{ik}, t_{jk})$ to indicate that $G_1 \in \mathfrak{M}_i$, $G_2 \in \mathfrak{M}_j$, $G_3 \in \mathfrak{M}_k$ with $|X_i \cap X_j| = t_{ij}$,$|X_i \cap X_k| = t_{ik}$ and $|X_j \cap X_k| = t_{jk}$. (Here $G_1 = Stab_G(X_i)$, $G_2 = Stab_G(X_j)$, $G_3 = Stab_G(X_k)$ for suitable X_i, X_j and X_k of $\Lambda \in \mathfrak{X}$). Again we run into the possibility that in some instances, we need further subdivide these cases, and we do this using the ad hoc notation $\mathfrak{M}_{ijk}(t_{ij}, t_{ik}, t_{jk} : l)$ where $l \in \{1, 2, 3, 4\}$. We note that if two or more i, j and k are equal, apparently different parameters t_{ij}, t_{ik}, t_{jk} may describe the same situation. For examle $\mathfrak{M}_{344}(2, 0, 4)$ and $\mathfrak{M}_{344}(0, 2, 4)$ describe the same configuration as do $\mathfrak{M}_{333}(3, 1, 1)$ and $\mathfrak{M}_{333}(1, 3, 1)$.

We remark that the geometry $\Gamma(G, \mathcal{F})$ where $\mathcal{F} = \{G_1, G_2, G_3\}$ is residually connected if and only if $G_1 =< G_{12}, G_{13} >$, $G_2 =< G_{12}, G_{23} >$ and $G_3 =< G_{13}, G_{23} >$.

Below we give certain subsets of Λ which will be encountered frequently in our list.

H_1=  O_2=  D_1= 

S_1= 

Our notation is as in [3] and [10] with the following addition: F_n a Frobenious group of order n and $(S_n \times S_m)^+$ is the group of even permutation in the permutation group $S_n \times S_m$.

2. Rank 3 geometries of M_{23}

Theorem 2. *Up to conjugacy in AutG there are* 1050 *rank* 3 *residually connected geometries of* $\Gamma = \Gamma(G, \{G_1, G_2, G_3\})$ *with* $G_1, G_2, G_3 \in \mathfrak{M}$. *These together with the shape of* G_{123} *are listed in the following table.*

Γ	G_{123}		Γ	G_{123}
$\mathfrak{M}_{111}(0,0,0)$	$2^4.SL(2,4)$		$\mathfrak{M}_{112}(0,0,1)$	$2^4.SL(2,4)$
$\mathfrak{M}_{112}(0,0,0)$	$2^4.S_3$		$\mathfrak{M}_{113}(0,1,1)$	$2^4.A_5$
$\mathfrak{M}_{113}(0,1,0)$	A_6		$\mathfrak{M}_{113}(0,0,0)$	$L_3(2)$
$\mathfrak{M}_{114}(0,1,1)$	A_6		$\mathfrak{M}_{114}(0,0,0)$	$2^4.S_3$
$\mathfrak{M}_{114}(0,0,1)$	$L_3(2)$		$\mathfrak{M}_{115}(0,1,1)$	A_5
$\mathfrak{M}_{115}(0,1,0)$	A_5		$\mathfrak{M}_{115}(0,0,0)$	$3^2.Q_8$
$\mathfrak{M}_{116}(0,1,1)$	$3^2.2$		$\mathfrak{M}_{116}(0,2,2)$	$2^4.A_5$
$\mathfrak{M}_{117}(0,1,1)$	1			
$\mathfrak{M}_{122}(1,0,0)$	$2^4.S_3$		$\mathfrak{M}_{122}(0,0,0:1)$	$2^4.2^2$
$\mathfrak{M}_{122}(0,0,0:2)$	D_{12}		$\mathfrak{M}_{122}(0,0,1)$	$2^4.3$
$\mathfrak{M}_{123}(1,1,2)$	$2^4.A_5$		$\mathfrak{M}_{123}(0,1,2)$	$2^4.S_4$
$\mathfrak{M}_{123}(1,0,0)$	$L_3(2)$		$\mathfrak{M}_{123}(0,0,2)$	S_5
$\mathfrak{M}_{123}(0,1,1)$	A_5		$\mathfrak{M}_{123}(0,1,0)$	$2^3.S_3$
$\mathfrak{M}_{123}(0,0,0)$	S_4		$\mathfrak{M}_{123}(0,0,1)$	S_4
$\mathfrak{M}_{124}(1,1,2)$	A_6		$\mathfrak{M}_{124}(0,1,2)$	S_5
$\mathfrak{M}_{124}(1,0,0)$	$2^4.S_3$		$\mathfrak{M}_{124}(0,0,2)$	$2^2.D_{12}$
$\mathfrak{M}_{124}(0,1,0)$	S_4		$\mathfrak{M}_{124}(0,1,1)$	S_4
$\mathfrak{M}_{124}(0,0,0)$	$2^3.2$		$\mathfrak{M}_{124}(0,0,1)$	A_4
$\mathfrak{M}_{125}(1,1,0)$	M_9		$\mathfrak{M}_{125}(1,0,2)$	A_5
$\mathfrak{M}_{125}(0,1,2:1)$	S_4		$\mathfrak{M}_{125}(0,1,2:2)$	F_{20}
$\mathfrak{M}_{125}(0,1,0)$	$Q_8.2$		$\mathfrak{M}_{125}(0,0,0)$	D_{12}
$\mathfrak{M}_{125}(0,0,2)$	D_{12}		$\mathfrak{M}_{125}(0,0,1:1)$	A_4
$\mathfrak{M}_{125}(0,0,1:2)$	D_{10}		$\mathfrak{M}_{125}(0,1,1)$	S_3
$\mathfrak{M}_{126}(1,2,3)$	$2^4.A_5$		$\mathfrak{M}_{126}(1,1,3)$	$2^4.S_3$
$\mathfrak{M}_{126}(0,1,3)$	$2^4.S_3$		$\mathfrak{M}_{126}(0,2,1)$	$2^3.D_8$
$\mathfrak{M}_{126}(0,2,4)$	$2^4.3$		$\mathfrak{M}_{126}(1,1,2)$	$3^2.2$
$\mathfrak{M}_{126}(0,1,1)$	D_{12}		$\mathfrak{M}_{126}(0,2,2)$	D_{12}
$\mathfrak{M}_{126}(0,1,2:1)$	D_{12}		$\mathfrak{M}_{126}(0,1,2:2)$	S_3
$\mathfrak{M}_{126}(0,1,2:3)$	2^2		$\mathfrak{M}_{126}(0,1,4)$	S_3
$\mathfrak{M}_{133}(1,1,3)$	$2^4.S_3$		$\mathfrak{M}_{133}(0,1,3)$	$(S_3 \times S_4)^+$
$\mathfrak{M}_{133}(1,0,1)$	A_5		$\mathfrak{M}_{133}(0,0,1)$	$3^2.4$
$\mathfrak{M}_{133}(0,0,3)$	S_4			
$\mathfrak{M}_{134}(1,0,0)$	$2^3.S_4$		$\mathfrak{M}_{134}(0,0,0)$	$L_3(2)$
$\mathfrak{M}_{134}(0,1,0)$	$L_3(2)$		$\mathfrak{M}_{134}(1,0,4)$	$2^2.S_4$
			$\mathfrak{M}_{134}(1,1,4)$	$(S_3 \times S_4)^+$
$\mathfrak{M}_{134}(0,1,4)$	$(S_3 \times S_4)^+$		$\mathfrak{M}_{134}(1,1,2)$	A_5
$\mathfrak{M}_{134}(0,0,4)$	S_4		$\mathfrak{M}_{134}(1,0,2)$	S_4
$\mathfrak{M}_{134}(0,1,2)$	F_{20}		$\mathfrak{M}_{134}(0,0,2)$	D_{12}
$\mathfrak{M}_{135}(1,0,6)$	A_5		$\mathfrak{M}_{135}(0,0,6)$	A_5
$\mathfrak{M}_{135}(1,0,2)$	A_5		$\mathfrak{M}_{135}(0,1,6)$	$3^2.4$
$\mathfrak{M}_{135}(1,1,2)$	S_4		$\mathfrak{M}_{135}(0,1,2)$	F_{20}
$\mathfrak{M}_{135}(1,1,4)$	$M_8.2$		$\mathfrak{M}_{135}(1,0,4)$	D_{12}

$\mathfrak{M}_{135}(0,0,2)$	D_{12}		$\mathfrak{M}_{135}(0,0,4)$	S_3
$\mathfrak{M}_{135}(0,1,4)$	S_3			
$\mathfrak{M}_{136}(1,2,1)$	$2^4.S_4$		$\mathfrak{M}_{136}(0,1,1)$	$(S_3\times S_4)^+$
$\mathfrak{M}_{136}(1,2,4)$	A_5		$\mathfrak{M}_{136}(0,2,2)$	S_4
$\mathfrak{M}_{136}(0,2,3)$	S_4		$\mathfrak{M}_{136}(1,1,4)$	S_4
$\mathfrak{M}_{136}(1,1,3)$	$3^2.2$		$\mathfrak{M}_{136}(1,1,2)$	2^3
$\mathfrak{M}_{136}(0,1,2:1)$	D_8		$\mathfrak{M}_{136}(0,1,2:2)$	S_3
$\mathfrak{M}_{136}(0,1,4)$	D_8		$\mathfrak{M}_{136}(0,1,3)$	S_3
$\mathfrak{M}_{144}(0,0,0)$	$2^3.S_4$		$\mathfrak{M}_{144}(1,0,0)$	$L_3(2)$
$\mathfrak{M}_{144}(0,0,4:1)$	$2^2.D_8$		$\mathfrak{M}_{144}(0,0,4:2)$	A_4
$\mathfrak{M}_{144}(1,1,2)$	$3^2.4$		$\mathfrak{M}_{144}(0,1,4)$	S_4
$\mathfrak{M}_{144}(1,1,4)$	S_4		$\mathfrak{M}_{144}(0,1,2)$	D_{12}
$\mathfrak{M}_{144}(0,0,2)$	D_8			
$\mathfrak{M}_{145}(1,0,2)$	A_5		$\mathfrak{M}_{145}(1,1,6)$	$3^2.4$
$\mathfrak{M}_{145}(0,1,2)$	S_4		$\mathfrak{M}_{145}(1,1,2)$	F_{20}
$\mathfrak{M}_{145}(0,0,2)$	D_{12}		$\mathfrak{M}_{145}(0,0,6)$	D_{12}
$\mathfrak{M}_{145}(1,0,6)$	D_{12}		$\mathfrak{M}_{145}(0,0,4:1)$	A_4
$\mathfrak{M}_{145}(0,0,4:2)$	2^2		$\mathfrak{M}_{145}(0,1,6)$	D_8
$\mathfrak{M}_{145}(1,0,4)$	S_3		$\mathfrak{M}_{145}(1,1,4)$	S_3
$\mathfrak{M}_{145}(0,1,4)$	4			
$\mathfrak{M}_{146}(0,2,1)$	$2^4.D_{12}$		$\mathfrak{M}_{146}(1,2,4)$	S_5
$\mathfrak{M}_{146}(0,1,1)$	$2^4.S_3$		$\mathfrak{M}_{146}(1,1,1)$	$(S_3\times S_4)^+$
$\mathfrak{M}_{146}(1,1,4)$	$(S_3\times S_4)^+$		$\mathfrak{M}_{146}(0,1,4)$	S_4
$\mathfrak{M}_{146}(1,2,5)$	S_4		$\mathfrak{M}_{146}(0,2,2)$	$2^3.2$
$\mathfrak{M}_{146}(0,2,3)$	A_4		$\mathfrak{M}_{146}(1,1,5)$	D_8
$\mathfrak{M}_{146}(0,1,5:1)$	2^3		$\mathfrak{M}_{146}(1,1,2)$	S_3
$\mathfrak{M}_{146}(0,1,5:2)$	S_3		$\mathfrak{M}_{146}(0,1,2)$	S_3
$\mathfrak{M}_{146}(1,1,3)$	S_3		$\mathfrak{M}_{146}(0,1,3)$	2
$\mathfrak{M}_{155}(1,1,4)$	$M_8.2$		$\mathfrak{M}_{155}(0,0,4)$	D_{12}
$\mathfrak{M}_{155}(0,0,8:1)$	A_4		$\mathfrak{M}_{155}(0,0,8:2)$	2^2
$\mathfrak{M}_{155}(1,0,4)$	S_3		$\mathfrak{M}_{155}(0,1,8)$	S_3
$\mathfrak{M}_{155}(1,1,8)$	4		$\mathfrak{M}_{155}(1,0,6)$	2
$\mathfrak{M}_{155}(1,1,6)$	2		$\mathfrak{M}_{155}(0,0,6)$	2
$\mathfrak{M}_{156}(1,1,1:1)$	$3^2.2$		$\mathfrak{M}_{156}(1,1,1:2)$	2^2
$\mathfrak{M}_{156}(1,2,5)$	$M_8.2$		$\mathfrak{M}_{156}(0,2,1)$	D_{12}
$\mathfrak{M}_{156}(0,1,1:1)$	D_{12}		$\mathfrak{M}_{156}(0,1,1:2)$	S_3
$\mathfrak{M}_{156}(0,2,4)$	A_4		$\mathfrak{M}_{156}(0,2,3)$	D_{10}
$\mathfrak{M}_{156}(1,2,2)$	S_3		$\mathfrak{M}_{156}(0,1,2:1)$	S_3
$\mathfrak{M}_{156}(0,1,2:2)$	2^2		$\mathfrak{M}_{156}(0,1,2:3)$	2
$\mathfrak{M}_{156}(0,1,5)$	S_3		$\mathfrak{M}_{156}(1,1,5)$	S_3
$\mathfrak{M}_{156}(1,1,2:1)$	2^2		$\mathfrak{M}_{156}(1,1,2:2)$	2
$\mathfrak{M}_{156}(0,1,4)$	2^2		$\mathfrak{M}_{156}(1,1,3)$	4

$\mathfrak{M}_{156}(1,1,4)$	4		$\mathfrak{M}_{156}(0,1,3)$	2
$\mathfrak{M}_{157}(0,1,1)$	1		$\mathfrak{M}_{157}(1,1,1)$	1
$\mathfrak{M}_{166}(2,2,4)$	$2^4.3$		$\mathfrak{M}_{166}(1,1,2:1)$	$3^2.2$
$\mathfrak{M}_{166}(1,1,2:2)$	S_3		$\mathfrak{M}_{166}(1,1,2:3)$	2^2
$\mathfrak{M}_{166}(1,1,5)$	$3^2.2$		$\mathfrak{M}_{166}(1,2,2)$	D_{12}
$\mathfrak{M}_{166}(1,1,4)$	S_3		$\mathfrak{M}_{166}(2,1,1:1)$	S_3
$\mathfrak{M}_{166}(2,1,1:2)$	2^2		$\mathfrak{M}_{166}(1,1,3)$	2^2
$\mathfrak{M}_{166}(1,1,1)$	2			
$\mathfrak{M}_{177}(1,1,2)$	1			
************	*****		************	*****
$\mathfrak{M}_{222}(0,0,0:1)$	2^3		$\mathfrak{M}_{222}(0,0,0:2)$	2^2
$\mathfrak{M}_{222}(0,1,0)$	S_3			
$\mathfrak{M}_{223}(1,2,2)$	$2^4.A_4$		$\mathfrak{M}_{223}(0,2,1)$	S_4
$\mathfrak{M}_{223}(0,2,0)$	$2^3.2$		$\mathfrak{M}_{223}(1,0,0)$	A_4
$\mathfrak{M}_{223}(0,1,0:1)$	D_8		$\mathfrak{M}_{223}(0,1,0:2)$	S_3
$\mathfrak{M}_{223}(0,0,0)$	2^2		$\mathfrak{M}_{223}(0,1,1)$	2^2
$\mathfrak{M}_{224}(1,2,2)$	A_5		$\mathfrak{M}_{224}(0,2,2)$	$2\times S_4$
$\mathfrak{M}_{224}(0,0,2:1)$	$D_8\times 2$		$\mathfrak{M}_{224}(0,2,0:2)$	D_{12}
$\mathfrak{M}_{224}(0,0,0:1)$	$2\times D_8$		$\mathfrak{M}_{224}(0,0,0:2)$	2^2
$\mathfrak{M}_{224}(0,1,2)$	D_8		$\mathfrak{M}_{224}(1,0,0)$	2^3
$\mathfrak{M}_{224}(0,1,1:1)$	2^2		$\mathfrak{M}_{224}(0,1,1:2)$	3
$\mathfrak{M}_{224}(0,0,1)$	2			
$\mathfrak{M}_{225}(0,2,2:1)$	D_8		$\mathfrak{M}_{225}(0,2,2:2)$	2^2
$\mathfrak{M}_{225}(0,2,0:1)$	D_8		$\mathfrak{M}_{225}(0,2,0:2)$	4
$\mathfrak{M}_{225}(1,0,0)$	Q_8		$\mathfrak{M}_{225}(1,2,2)$	S_3
$\mathfrak{M}_{225}(0,0,0)$	2^2		$\mathfrak{M}_{225}(0,1,1)$	3
$\mathfrak{M}_{225}(0,0,1)$	2		$\mathfrak{M}_{225}(0,2,1)$	2
$\mathfrak{M}_{225}(0,1,1)$	2			
$\mathfrak{M}_{226}(0,3,2)$	D_{12}		$\mathfrak{M}_{226}(0,1,1)$	2^3
$\mathfrak{M}_{226}(0,1,2:1)$	S_3		$\mathfrak{M}_{226}(0,1,2:2)$	2^2
$\mathfrak{M}_{226}(1,1,2)$	S_3		$\mathfrak{M}_{226}(0,2,4:1)$	2^2
$\mathfrak{M}_{226}(0,2,4:2)$	2		$\mathfrak{M}_{226}(0,1,4)$	2^2
$\mathfrak{M}_{226}(0,4,4)$	3		$\mathfrak{M}_{226}(0,2,2:1)$	2
$\mathfrak{M}_{226}(0,2,2:2)$	1		$\mathfrak{M}_{226}(1,2,2)$	2
$\mathfrak{M}_{233}(2,2,3)$	$2^4.S_3$		$\mathfrak{M}_{233}(2,0,3)$	$S_4.2$
$\mathfrak{M}_{233}(2,0,1)$	S_4		$\mathfrak{M}_{233}(2,1,3)$	S_4
$\mathfrak{M}_{233}(0,0,3:1)$	$2^3.2$		$\mathfrak{M}_{233}(0,0,3:2)$	S_3
$\mathfrak{M}_{233}(0,0,1)$	D_8		$\mathfrak{M}_{233}(1,0,3)$	S_3
$\mathfrak{M}_{233}(1,0,1)$	S_3			
$\mathfrak{M}_{234}(2,0,0)$	$2^3.D_8$		$\mathfrak{M}_{234}(2,2,4)$	$S_4\times 2$
$\mathfrak{M}_{234}(0,2,4)$	$S_4\times 2$		$\mathfrak{M}_{234}(0,0,0)$	$S_4.2$

$\mathfrak{M}_{234}(0,2,0)$	$S_4.2$		$\mathfrak{M}_{234}(2,1,4)$	S_4
$\mathfrak{M}_{234}(1,0,0)$	S_4		$\mathfrak{M}_{234}(0,1,0)$	F_{21}
$\mathfrak{M}_{234}(1,2,4)$	$3^2.2$		$\mathfrak{M}_{234}(0,0,4:1)$	$2 \times D_8$
$\mathfrak{M}_{234}(0,0,4:2)$	S_3		$\mathfrak{M}_{234}(2,1,2)$	A_4
$\mathfrak{M}_{234}(2,0,2)$	D_{12}		$\mathfrak{M}_{234}(1,2,2)$	D_{10}
$\mathfrak{M}_{234}(0,2,2)$	D_8		$\mathfrak{M}_{234}(1,0,4:1)$	D_8
$\mathfrak{M}_{234}(1,0,4:2)$	S_3		$\mathfrak{M}_{234}(0,0,2:1)$	D_8
$\mathfrak{M}_{234}(0,0,2:2)$	2^2		$\mathfrak{M}_{234}(0,1,4)$	S_3
$\mathfrak{M}_{234}(1,0,2:1)$	S_3		$\mathfrak{M}_{234}(1,0,2:2)$	2^2
$\mathfrak{M}_{234}(0,1,2)$	2			
$\mathfrak{M}_{235}(2,2,6)$	S_4		$\mathfrak{M}_{235}(0,2,6)$	S_4
$\mathfrak{M}_{235}(2,0,4)$	D_{16}		$\mathfrak{M}_{235}(2,1,2)$	A_4
$\mathfrak{M}_{235}(1,2,6)$	D_{10}		$\mathfrak{M}_{235}(0,0,6)$	D_8
$\mathfrak{M}_{235}(0,0,2)$	D_8		$\mathfrak{M}_{235}(2,2,4)$	D_8
$\mathfrak{M}_{235}(0,2,2:1)$	D_8		$\mathfrak{M}_{235}(0,2,2:2)$	2^2
$\mathfrak{M}_{235}(1,2,2)$	S_3		$\mathfrak{M}_{235}(1,2,4:1)$	S_3
$\mathfrak{M}_{235}(1,2,4:2)$	2		$\mathfrak{M}_{235}(0,1,6)$	S_3
$\mathfrak{M}_{235}(1,0,2)$	4		$\mathfrak{M}_{235}(2,1,4)$	2^2
$\mathfrak{M}_{235}(0,1,4:1)$	3		$\mathfrak{M}_{235}(0,1,4:2)$	2
$\mathfrak{M}_{235}(0,1,2)$	2		$\mathfrak{M}_{235}(0,2,4)$	2
$\mathfrak{M}_{235}(0,0,4)$	2		$\mathfrak{M}_{235}(1,0,4)$	2
$\mathfrak{M}_{236}(2,3,1)$	$2^4.S_4$		$\mathfrak{M}_{236}(0,1,1)$	$S_4.2$
$\mathfrak{M}_{236}(0,3,3)$	S_4		$\mathfrak{M}_{236}(1,2,1)$	$3^2.2$
$\mathfrak{M}_{236}(2,1,2)$	$2 \times D_8$		$\mathfrak{M}_{236}(2,4,4)$	A_4
$\mathfrak{M}_{236}(2,2,4)$	D_{12}		$\mathfrak{M}_{236}(0,2,1)$	D_{12}
$\mathfrak{M}_{236}(0,1,4)$	D_8		$\mathfrak{M}_{236}(1,1,4)$	D_8
$\mathfrak{M}_{236}(2,2,3)$	S_3		$\mathfrak{M}_{236}(0,1,3)$	S_3
$\mathfrak{M}_{236}(1,1,3)$	S_3		$\mathfrak{M}_{236}(0,2,3:1)$	S_3
$\mathfrak{M}_{236}(0,2,3:2)$	2^2		$\mathfrak{M}_{236}(0,2,3:3)$	2
$\mathfrak{M}_{236}(0,4,2:1)$	2^2		$\mathfrak{M}_{236}(0,4,2:2)$	3
$\mathfrak{M}_{236}(2,2,2)$	2^2		$\mathfrak{M}_{236}(0,2,4:1)$	2^2
$\mathfrak{M}_{236}(0,2,4:2)$	2		$\mathfrak{M}_{236}(0,1,2)$	2^2
$\mathfrak{M}_{236}(1,1,2)$	2^2		$\mathfrak{M}_{236}(1,2,3)$	2
$\mathfrak{M}_{236}(1,2,2)$	2		$\mathfrak{M}_{236}(0,4,3)$	2
$\mathfrak{M}_{236}(1,2,4)$	2		$\mathfrak{M}_{236}(0,2,2)$	1
$\mathfrak{M}_{244}(2,0,0)$	$S_4 \times 2$		$\mathfrak{M}_{244}(1,0,0)$	S_4
$\mathfrak{M}_{244}(2,2,4)$	$2^3.2$		$\mathfrak{M}_{244}(2,0,4)$	$2^3.2$
$\mathfrak{M}_{244}(2,0,2:1)$	D_{12}		$\mathfrak{M}_{244}(2,0,2:2)$	D_8
$\mathfrak{M}_{244}(0,0,4:1)$	2^3		$\mathfrak{M}_{244}(0,0,4:2)$	S_3
$\mathfrak{M}_{244}(0,0,4:3)$	2^2		$\mathfrak{M}_{244}(1,0,4:1)$	D_8
$\mathfrak{M}_{244}(0,1,4:2)$	3		$\mathfrak{M}_{244}(2,1,2)$	S_3
$\mathfrak{M}_{244}(1,2,4)$	S_3		$\mathfrak{M}_{244}(1,0,2:1)$	2^2

$\mathfrak{M}_{244}(1,0,2:2)$	2		$\mathfrak{M}_{244}(0,0,2)$	2^2
$\mathfrak{M}_{244}(1,1,2)$	2			

$\mathfrak{M}_{245}(2,2,6:1)$	D_{12}		$\mathfrak{M}_{245}(2,2,6:2)$	D_8
$\mathfrak{M}_{245}(0,2,6:1)$	D_{12}		$\mathfrak{M}_{245}(0,2,6:2)$	D_8
$\mathfrak{M}_{245}(0,0,2)$	D_{12}		$\mathfrak{M}_{245}(2,1,2)$	D_{10}
$\mathfrak{M}_{245}(0,2,2:1)$	D_8		$\mathfrak{M}_{245}(0,2,2:2)$	2^2
$\mathfrak{M}_{245}(2,0,2)$	D_8		$\mathfrak{M}_{245}(2,1,4:1)$	S_3
$\mathfrak{M}_{245}(2,1,4:2)$	2		$\mathfrak{M}_{245}(1,2,2)$	S_3
$\mathfrak{M}_{245}(2,1,6)$	S_3		$\mathfrak{M}_{245}(0,1,2:1)$	S_3
$\mathfrak{M}_{245}(0,1,2:2)$	2^2		$\mathfrak{M}_{245}(0,0,4:1)$	4
$\mathfrak{M}_{245}(0,0,4:2)$	2		$\mathfrak{M}_{245}(1,0,2)$	4
$\mathfrak{M}_{245}(1,0,6)$	4		$\mathfrak{M}_{245}(1,0,4:1)$	4
$\mathfrak{M}_{245}(1,0,4:2)$	1		$\mathfrak{M}_{245}(2,0,4)$	2^2
$\mathfrak{M}_{245}(0,0,6)$	2^2		$\mathfrak{M}_{245}(0,1,6:1)$	2^2
$\mathfrak{M}_{245}(0,1,6:2)$	2		$\mathfrak{M}_{245}(2,2,4)$	2^2
$\mathfrak{M}_{245}(1,2,4:1)$	3		$\mathfrak{M}_{245}(1,2,4:2)$	2
$\mathfrak{M}_{245}(0,1,4:1)$	2		$\mathfrak{M}_{245}(0,1,4:2)$	1
$\mathfrak{M}_{245}(1,2,6)$	2		$\mathfrak{M}_{245}(0,2,4)$	2

$\mathfrak{M}_{246}(2,3,4)$	S_5		$\mathfrak{M}_{246}(2,1,1)$	$S_4.2$
$\mathfrak{M}_{246}(0,1,1)$	$2^3.2^2$		$\mathfrak{M}_{246}(2,4,4)$	S_4
$\mathfrak{M}_{246}(0,1,4)$	S_4		$\mathfrak{M}_{246}(1,1,4)$	S_4
$\mathfrak{M}_{246}(0,4,1)$	$2^3.2$		$\mathfrak{M}_{246}(0,3,2)$	$2^3.2$
$\mathfrak{M}_{246}(2,1,5)$	$D_8 \times 2$		$\mathfrak{M}_{246}(0,2,1)$	D_{12}
$\mathfrak{M}_{246}(2,1,2)$	D_{12}		$\mathfrak{M}_{246}(1,2,4)$	S_3
$\mathfrak{M}_{246}(1,2,1)$	S_3		$\mathfrak{M}_{246}(0,2,2:1)$	S_3
$\mathfrak{M}_{246}(0,2,2:2)$	2^2		$\mathfrak{M}_{246}(0,2,2:3)$	2
$\mathfrak{M}_{246}(0,2,4)$	2^2		$\mathfrak{M}_{246}(2,2,3:1)$	2^2
$\mathfrak{M}_{246}(2,2,3:2)$	2		$\mathfrak{M}_{246}(0,1,5)$	2^2
$\mathfrak{M}_{246}(1,1,5)$	2^2		$\mathfrak{M}_{246}(2,2,2)$	2^2
$\mathfrak{M}_{246}(2,4,5)$	2^2		$\mathfrak{M}_{246}(2,2,5)$	2^2
$\mathfrak{M}_{246}(1,1,2)$	3		$\mathfrak{M}_{246}(0,4,2)$	2
$\mathfrak{M}_{246}(1,2,3:1)$	2		$\mathfrak{M}_{246}(1,2,3:2)$	1
$\mathfrak{M}_{246}(0,1,3)$	2		$\mathfrak{M}_{246}(1,2,5:1)$	2
$\mathfrak{M}_{246}(1,2,5:2)$	1		$\mathfrak{M}_{246}(1,2,2:1)$	2
$\mathfrak{M}_{246}(1,2,2:2)$	1		$\mathfrak{M}_{246}(0,2,3:1)$	2
$\mathfrak{M}_{246}(0,2,3:2)$	1		$\mathfrak{M}_{246}(1,1,3)$	2
$\mathfrak{M}_{246}(0,2,5)$	2		$\mathfrak{M}_{246}(0,4,3)$	1

$\mathfrak{M}_{255}(0,0,4)$	D_{16}		$\mathfrak{M}_{255}(0,0,8:1)$	D_8
$\mathfrak{M}_{255}(0,0,8:2)$	4		$\mathfrak{M}_{255}(0,0,8:3)$	2
$\mathfrak{M}_{255}(2,2,4)$	D_8		$\mathfrak{M}_{255}(2,0,8)$	4
$\mathfrak{M}_{255}(2,1,8:1)$	3		$\mathfrak{M}_{255}(2,1,8:2)$	2
$\mathfrak{M}_{255}(2,2,8)$	2		$\mathfrak{M}_{255}(0,2,6)$	2

$\mathfrak{M}_{255}(0,1,6:1)$	2		$\mathfrak{M}_{255}(0,1,6:2)$	1
$\mathfrak{M}_{255}(0,0,6)$	2		$\mathfrak{M}_{255}(2,0,4)$	2
$\mathfrak{M}_{255}(1,2,6:1)$	2		$\mathfrak{M}_{255}(1,2,6:2)$	1
$\mathfrak{M}_{255}(2,2,6)$	2		$\mathfrak{M}_{255}(1,0,4)$	2
$\mathfrak{M}_{255}(2,1,4)$	2		$\mathfrak{M}_{255}(1,0,8)$	1
$\mathfrak{M}_{256}(0,3,5)$	$Q_8.2$		$\mathfrak{M}_{256}(2,3,1)$	D_{12}
$\mathfrak{M}_{256}(0,1,5)$	D_{12}		$\mathfrak{M}_{256}(0,1,4)$	D_8
$\mathfrak{M}_{256}(2,1,4)$	D_8		$\mathfrak{M}_{256}(2,1,1)$	S_3
$\mathfrak{M}_{256}(1,2,5:1)$	S_3		$\mathfrak{M}_{256}(1,2,5:2)$	2
$\mathfrak{M}_{256}(0,2,1:1)$	2^2		$\mathfrak{M}_{256}(0,2,1:2)$	2
$\mathfrak{M}_{256}(1,1,2:1)$	2^2		$\mathfrak{M}_{256}(1,1,2:2)$	2
$\mathfrak{M}_{256}(2,1,2:1)$	2^2		$\mathfrak{M}_{256}(2,1,2:2)$	2
$\mathfrak{M}_{256}(2,4,1:1)$	2^2		$\mathfrak{M}_{256}(2,4,1:2)$	2
$\mathfrak{M}_{256}(1,1,1)$	2^2		$\mathfrak{M}_{256}(0,1,1)$	2^2
$\mathfrak{M}_{256}(2,1,3)$	4		$\mathfrak{M}_{256}(0,1,3)$	4
$\mathfrak{M}_{256}(1,1,5)$	3		$\mathfrak{M}_{256}(2,2,1)$	2
$\mathfrak{M}_{256}(1,2,1:1)$	2		$\mathfrak{M}_{256}(1,2,1:2)$	1
$\mathfrak{M}_{256}(1,2,2:1)$	2		$\mathfrak{M}_{256}(1,2,2:2)$	1
$\mathfrak{M}_{256}(0,1,2)$	2		$\mathfrak{M}_{256}(0,2,2:1)$	2
$\mathfrak{M}_{256}(0,2,2:2)$	1		$\mathfrak{M}_{256}(2,2,3:1)$	2
$\mathfrak{M}_{256}(2,2,3:2)$	1		$\mathfrak{M}_{256}(0,2,4:1)$	2
$\mathfrak{M}_{256}(0,2,4:2)$	1		$\mathfrak{M}_{256}(1,2,4:1)$	2
$\mathfrak{M}_{256}(1,2,4:2)$	1		$\mathfrak{M}_{256}(0,2,5)$	2
$\mathfrak{M}_{256}(1,1,3)$	2		$\mathfrak{M}_{256}(2,4,3)$	2
$\mathfrak{M}_{256}(1,1,4)$	2		$\mathfrak{M}_{256}(2,2,4)$	2
$\mathfrak{M}_{256}(2,4,4)$	2		$\mathfrak{M}_{256}(2,2,5)$	2
$\mathfrak{M}_{256}(0,4,5)$	2		$\mathfrak{M}_{256}(0,4,2:1)$	2
$\mathfrak{M}_{256}(0,4,2:2)$	1		$\mathfrak{M}_{256}(0,2,3)$	1
$\mathfrak{M}_{256}(2,2,2)$	1		$\mathfrak{M}_{256}(1,2,3)$	1
$\mathfrak{M}_{266}(2,1,5)$	D_{12}		$\mathfrak{M}_{266}(1,1,3)$	2^3
$\mathfrak{M}_{266}(1,1,2)$	S_3		$\mathfrak{M}_{266}(4,2,5)$	S_3
$\mathfrak{M}_{266}(1,1,1)$	2^2		$\mathfrak{M}_{266}(2,3,1)$	2^2
$\mathfrak{M}_{266}(2,1,3)$	2^2		$\mathfrak{M}_{266}(1,2,2)$	2^2
$\mathfrak{M}_{266}(1,4,2)$	2^2		$\mathfrak{M}_{266}(2,1,4)$	2^2
$\mathfrak{M}_{266}(2,2,5)$	2^2		$\mathfrak{M}_{266}(2,2,3)$	2
$\mathfrak{M}_{266}(2,4,3)$	2		$\mathfrak{M}_{266}(1,2,1)$	2
$\mathfrak{M}_{266}(1,4,1)$	2		$\mathfrak{M}_{266}(2,2,2:1)$	2
$\mathfrak{M}_{266}(2,2,2:2)$	1		$\mathfrak{M}_{266}(2,2,4)$	2
$\mathfrak{M}_{266}(4,2,2)$	2		$\mathfrak{M}_{266}(2,2,1)$	1
$\mathfrak{M}_{266}(2,1,1)$	1		$\mathfrak{M}_{266}(4,2,1)$	1
************	*****		************	*****
$\mathfrak{M}_{333}(3,3,1:1)$	$3^2.2$		$\mathfrak{M}_{333}(3,1,3:2)$	D_8

$\mathfrak{M}_{333}(1,1,1)$	D_{10}		$\mathfrak{M}_{333}(3,3,3:1)$	2^3
$\mathfrak{M}_{333}(3,3,3:2)$	S_3		$\mathfrak{M}_{333}(3,1,1)$	S_3
$\mathfrak{M}_{334}(3,0,0)$	$2^4.S_3$		$\mathfrak{M}_{334}(1,0,4)$	S_4
$\mathfrak{M}_{334}(1,0,2)$	S_4		$\mathfrak{M}_{334}(3,4,4:1)$	$3^2.2$
$\mathfrak{M}_{334}(3,4,4:2)$	2^3		$\mathfrak{M}_{334}(1,4,4)$	$3^2.2$
$\mathfrak{M}_{334}(3,0,2)$	D_{12}		$\mathfrak{M}_{334}(1,2,2:1)$	D_{10}
$\mathfrak{M}_{334}(1,2,2:2)$	2		$\mathfrak{M}_{334}(3,2,4:1)$	D_8
$\mathfrak{M}_{334}(3,2,4:2)$	S_3		$\mathfrak{M}_{334}(1,2,4:1)$	D_8
$\mathfrak{M}_{334}(1,2,4:2)$	S_3		$\mathfrak{M}_{334}(3,2,2:1)$	2^2
$\mathfrak{M}_{334}(3,2,2:2)$	2			
$\mathfrak{M}_{335}(3,2,6)$	S_4		$\mathfrak{M}_{335}(3,6,6)$	$3^2.2$
$\mathfrak{M}_{335}(1,2,6)$	D_{10}		$\mathfrak{M}_{335}(3,4,6:1)$	D_8
$\mathfrak{M}_{335}(3,4,6:2)$	S_3		$\mathfrak{M}_{335}(3,2,4:1)$	D_8
$\mathfrak{M}_{335}(3,2,4:2)$	2		$\mathfrak{M}_{335}(1,2,2)$	D_8
$\mathfrak{M}_{335}(1,4,6)$	S_3		$\mathfrak{M}_{335}(3,4,4:1)$	S_3
$\mathfrak{M}_{335}(3,4,4:2)$	2^2		$\mathfrak{M}_{335}(3,4,4:3)$	2
$\mathfrak{M}_{335}(3,4,4:4)$	1		$\mathfrak{M}_{335}(1,2,4:1)$	S_3
$\mathfrak{M}_{335}(1,2,4:2)$	2		$\mathfrak{M}_{335}(3,2,2:1)$	S_3
$\mathfrak{M}_{335}(3,2,2:2)$	2^2		$\mathfrak{M}_{335}(1,4,4)$	2
$\mathfrak{M}_{336}(3,4,1)$	S_4		$\mathfrak{M}_{336}(1,3,1)$	$3^2.2$
$\mathfrak{M}_{336}(3,2,1)$	$2^3.2$		$\mathfrak{M}_{336}(1,4,4)$	D_{10}
$\mathfrak{M}_{336}(1,3,3)$	S_3		$\mathfrak{M}_{336}(3,3,2:1)$	S_3
$\mathfrak{M}_{336}(3,3,2:2)$	2^2		$\mathfrak{M}_{336}(3,3,2:3)$	2
$\mathfrak{M}_{336}(1,4,3:1)$	S_3		$\mathfrak{M}_{336}(1,4,3:2)$	4
$\mathfrak{M}_{336}(3,4,4)$	S_3		$\mathfrak{M}_{336}(1,3,2:1)$	2^2
$\mathfrak{M}_{336}(1,3,2:2)$	2		$\mathfrak{M}_{336}(3,4,3)$	2^2
$\mathfrak{M}_{336}(1,2,2)$	2		$\mathfrak{M}_{336}(3,4,2)$	2
$\mathfrak{M}_{336}(3,3,3)$	2			
$\mathfrak{M}_{344}(4,0,0)$	$2^4.S_3$		$\mathfrak{M}_{344}(0,0,4)$	$2^4.3$
$\mathfrak{M}_{344}(2,4,0)$	S_4		$\mathfrak{M}_{344}(4,4,2:1)$	$3^2.2$
$\mathfrak{M}_{344}(4,4,2:2)$	D_8		$\mathfrak{M}_{344}(2,2,0)$	D_{12}
$\mathfrak{M}_{344}(0,4,2)$	D_{12}		$\mathfrak{M}_{344}(4,4,4:1)$	2^3
$\mathfrak{M}_{344}(4,4,4:2)$	S_3		$\mathfrak{M}_{344}(2,4,2:1)$	D_8
$\mathfrak{M}_{344}(4,2,2:2)$	2		$\mathfrak{M}_{344}(0,2,2)$	D_8
$\mathfrak{M}_{344}(2,0,4)$	D_8		$\mathfrak{M}_{344}(2,4,4:1)$	S_3
$\mathfrak{M}_{344}(2,4,4:2)$	2^2		$\mathfrak{M}_{344}(2,2,4:1)$	2^2
$\mathfrak{M}_{344}(2,2,4:2)$	3		$\mathfrak{M}_{344}(2,2,4:3)$	2
$\mathfrak{M}_{344}(2,2,2)$	2			
$\mathfrak{M}_{345}(0,6,2)$	S_4		$\mathfrak{M}_{345}(4,6,6:1)$	$3^2.2$
$\mathfrak{M}_{345}(4,6,6:2)$	D_8		$\mathfrak{M}_{345}(0,4,2)$	D_{12}
$\mathfrak{M}_{345}(4,2,6)$	D_{12}		$\mathfrak{M}_{345}(0,4,6)$	D_{12}
$\mathfrak{M}_{345}(2,6,2)$	D_{10}		$\mathfrak{M}_{345}(4,4,2:1)$	D_8

$\mathfrak{M}_{345}(4,4,2:2)$	S_3		$\mathfrak{M}_{345}(0,4,4:1)$	D_8
$\mathfrak{M}_{345}(0,4,4:2)$	3		$\mathfrak{M}_{345}(4,4,6:1)$	D_8
$\mathfrak{M}_{345}(4,4,6:2)$	2		$\mathfrak{M}_{345}(2,6,6)$	D_8
$\mathfrak{M}_{345}(0,2,6)$	D_8		$\mathfrak{M}_{345}(4,2,2:1)$	D_8
$\mathfrak{M}_{345}(4,2,2:2)$	S_3		$\mathfrak{M}_{345}(0,2,4)$	D_8
$\mathfrak{M}_{345}(2,6,4:1)$	S_3		$\mathfrak{M}_{345}(2,6,4:2)$	2
$\mathfrak{M}_{345}(4,4,4:1)$	S_3		$\mathfrak{M}_{345}(4,4,4:2)$	2^2
$\mathfrak{M}_{345}(4,4,4:3)$	2		$\mathfrak{M}_{345}(4,4,4:4)$	1
$\mathfrak{M}_{345}(4,6,4)$	S_3			
$\mathfrak{M}_{345}(4,2,4:1)$	2^2		$\mathfrak{M}_{345}(4,2,4:2)$	2
$\mathfrak{M}_{345}(2,2,2)$	2^2		$\mathfrak{M}_{345}(2,2,4:1)$	3
$\mathfrak{M}_{345}(2,2,4:2)$	2		$\mathfrak{M}_{345}(2,4,6)$	2
$\mathfrak{M}_{345}(2,4,2)$	2		$\mathfrak{M}_{345}(2,4,4:1)$	2
$\mathfrak{M}_{345}(2,4,4,:2)$	1		$\mathfrak{M}_{345}(2,2,6)$	2
$\mathfrak{M}_{346}(0,1,1)$	$2^4.S_3$		$\mathfrak{M}_{346}(4,1,4)$	$(S_3 \times S_4)^+$
$\mathfrak{M}_{346}(0,3,4)$	S_4		$\mathfrak{M}_{346}(4,3,1)$	$3^2.2$
$\mathfrak{M}_{346}(4,2,1)$	$2^3.2$		$\mathfrak{M}_{346}(4,1,5)$	$2^3.2$
$\mathfrak{M}_{346}(2,4,1)$	D_{12}		$\mathfrak{M}_{346}(2,1,2)$	D_{12}
$\mathfrak{M}_{346}(4,2,4)$	D_{12}		$\mathfrak{M}_{346}(4,4,4)$	D_{12}
$\mathfrak{M}_{346}(2,3,1)$	D_{12}		$\mathfrak{M}_{346}(0,4,2)$	D_8
$\mathfrak{M}_{346}(2,2,1)$	D_8		$\mathfrak{M}_{346}(4,4,2)$	D_8
$\mathfrak{M}_{346}(4,2,2:1)$	2^3		$\mathfrak{M}_{346}(0,2,2)$	2^3
$\mathfrak{M}_{346}(4,2,2:2)$	2			
$\mathfrak{M}_{346}(2,3,2:1)$	S_3		$\mathfrak{M}_{346}(2,3,2:2)$	2
$\mathfrak{M}_{346}(2,1,3)$	S_3		$\mathfrak{M}_{346}(0,3,3)$	S_3
$\mathfrak{M}_{346}(0,3,5)$	S_3		$\mathfrak{M}_{346}(4,4,5)$	S_3
$\mathfrak{M}_{346}(4,3,2:1)$	S_3		$\mathfrak{M}_{346}(4,3,2:2)$	2
$\mathfrak{M}_{346}(2,3,4)$	S_3		$\mathfrak{M}_{346}(2,4,2:1)$	2^2
$\mathfrak{M}_{346}(2,4,2:2)$	2		$\mathfrak{M}_{346}(2,2,2:1)$	2^2
$\mathfrak{M}_{346}(2,2,2:2)$	2		$\mathfrak{M}_{346}(2,2,4)$	2^2
$\mathfrak{M}_{346}(4,3,5)$	2^2		$\mathfrak{M}_{346}(2,3,5:1)$	2^2
$\mathfrak{M}_{346}(2,3,5:2)$	2			
$\mathfrak{M}_{346}(0,2,3)$	3		$\mathfrak{M}_{346}(2,4,3:1)$	3
$\mathfrak{M}_{346}(4,3,3)$	2		$\mathfrak{M}_{346}(2,4,3:2)$	2
$\mathfrak{M}_{346}(2,4,5)$	2		$\mathfrak{M}_{346}(2,3,3:1)$	2
$\mathfrak{M}_{346}(2,3,3:2)$	1		$\mathfrak{M}_{346}(4,4,3)$	2
$\mathfrak{M}_{346}(4,2,5)$	2		$\mathfrak{M}_{346}(2,2,3)$	1
$\mathfrak{M}_{355}(2,2,4)$	D_{12}		$\mathfrak{M}_{355}(2,6,4:1)$	D_8
$\mathfrak{M}_{355}(2,6,4:2)$	S_3		$\mathfrak{M}_{355}(6,4,4)$	S_3
$\mathfrak{M}_{355}(4,6,8:1)$	S_3		$\mathfrak{M}_{355}(4,6,8:2)$	2
$\mathfrak{M}_{355}(6,6,8)$	S_3		$\mathfrak{M}_{355}(2,2,8:1)$	2^2
$\mathfrak{M}_{355}(2,2,8:2)$	3		$\mathfrak{M}_{355}(2,2,8:3)$	2
$\mathfrak{M}_{355}(4,2,8)$	2		$\mathfrak{M}_{355}(4,4,8:1)$	2

$\mathfrak{M}_{355}(4,4,8:2)$	1		$\mathfrak{M}_{355}(2,6,6)$	2
$\mathfrak{M}_{355}(4,4,4:1)$	2		$\mathfrak{M}_{355}(4,4,4:2)$	1
$\mathfrak{M}_{355}(2,4,6:1)$	2		$\mathfrak{M}_{355}(4,2,6:2)$	1
$\mathfrak{M}_{355}(4,6,6)$	2		$\mathfrak{M}_{355}(2,2,6)$	2
$\mathfrak{M}_{355}(2,4,4)$	2		$\mathfrak{M}_{355}(4,4,6:1)$	2
$\mathfrak{M}_{355}(4,4,6:2)$	1			
$\mathfrak{M}_{356}(6,1,1)$	$3^2.2$		$\mathfrak{M}_{356}(4,1,1)$	D_{12}
$\mathfrak{M}_{356}(2,1,5)$	D_{12}		$\mathfrak{M}_{356}(2,2,5:1)$	D_8
$\mathfrak{M}_{356}(2,2,5:2)$	2		$\mathfrak{M}_{356}(4,1,4)$	D_8
$\mathfrak{M}_{356}(2,3,1:1)$	S_3		$\mathfrak{M}_{356}(2,3,1:2)$	2^2
$\mathfrak{M}_{356}(2,3,1:3)$	2		$\mathfrak{M}_{356}(2,1,2)$	S_3
$\mathfrak{M}_{356}(6,4,2)$	S_3		$\mathfrak{M}_{356}(4,3,1:1)$	S_3
$\mathfrak{M}_{356}(4,3,1:2)$	2		$\mathfrak{M}_{356}(2,3,5)$	S_3
$\mathfrak{M}_{356}(6,4,1)$	2^2		$\mathfrak{M}_{356}(6,2,1)$	2^2
$\mathfrak{M}_{356}(2,3,2:1)$	2^2		$\mathfrak{M}_{356}(2,3,2:2)$	2
$\mathfrak{M}_{356}(4,1,2)$	2^2		$\mathfrak{M}_{356}(4,1,3)$	4
$\mathfrak{M}_{356}(6,3,3)$	4		$\mathfrak{M}_{356}(2,4,4)$	3
$\mathfrak{M}_{356}(2,2,1)$	2		$\mathfrak{M}_{356}(4,4,1)$	2
$\mathfrak{M}_{356}(2,2,2:1)$	2		$\mathfrak{M}_{356}(2,2,2:2)$	1
$\mathfrak{M}_{356}(4,3,2:1)$	2		$\mathfrak{M}_{356}(4,3,2:2)$	1
$\mathfrak{M}_{356}(6,3,2)$	2		$\mathfrak{M}_{356}(4,3,3:1)$	2
$\mathfrak{M}_{356}(4,3,3:2)$	1		$\mathfrak{M}_{356}(2,4,3)$	2
$\mathfrak{M}_{356}(6,2,4)$	2		$\mathfrak{M}_{356}(2,3,4)$	2
$\mathfrak{M}_{356}(4,3,5)$	2		$\mathfrak{M}_{356}(4,2,5:1)$	2
$\mathfrak{M}_{356}(4,2,5:2)$	1		$\mathfrak{M}_{356}(4,4,5)$	2
$\mathfrak{M}_{356}(6,2,3)$	2		$\mathfrak{M}_{356}(4,2,3)$	2
$\mathfrak{M}_{356}(4,2,4:1)$	2		$\mathfrak{M}_{356}(4,2,4:2)$	1
$\mathfrak{M}_{356}(2,2,4)$	2		$\mathfrak{M}_{356}(2,4,5)$	2
$\mathfrak{M}_{356}(4,2,1:1)$	2		$\mathfrak{M}_{356}(4,2,1:2)$	1
$\mathfrak{M}_{356}(4,2,2)$	1		$\mathfrak{M}_{356}(4,4,2)$	1
$\mathfrak{M}_{356}(2,3,3)$	1		$\mathfrak{M}_{356}(4,3,4)$	1
$\mathfrak{M}_{356}(4,4,4)$	1			
$\mathfrak{M}_{366}(4,2,5)$	D_8		$\mathfrak{M}_{366}(2,3,5)$	S_3
$\mathfrak{M}_{366}(4,4,3)$	2^2			
$\mathfrak{M}_{366}(3,1,1)$	2^2		$\mathfrak{M}_{366}(2,2,2:1)$	2^2
$\mathfrak{M}_{366}(2,2,2:2)$	1		$\mathfrak{M}_{366}(3,3,2)$	2^2
$\mathfrak{M}_{366}(2,4,4)$	2^2		$\mathfrak{M}_{366}(4,4,2)$	2^2
$\mathfrak{M}_{366}(3,2,3)$	2		$\mathfrak{M}_{366}(3,3,3)$	2
$\mathfrak{M}_{366}(3,4,1:1)$	2		$\mathfrak{M}_{366}(3,4,1:2)$	1
$\mathfrak{M}_{366}(3,3,1:1)$	2		$\mathfrak{M}_{366}(3,3,1:2)$	1
$\mathfrak{M}_{366}(4,4,1)$	2		$\mathfrak{M}_{366}(2,4,2)$	2
$\mathfrak{M}_{366}(3,3,4)$	2		$\mathfrak{M}_{366}(3,4,2)$	2

$\mathfrak{M}_{366}(2,3,2)$	2		$\mathfrak{M}_{366}(3,2,1)$	1
$\mathfrak{M}_{366}(2,4,1)$	1		$\mathfrak{M}_{366}(2,2,1)$	1
∗ ∗ ∗ ∗ ∗ ∗ ∗ ∗ ∗ ∗ ∗∗	∗ ∗ ∗ ∗ ∗		∗ ∗ ∗ ∗ ∗ ∗ ∗ ∗ ∗ ∗ ∗∗	∗ ∗ ∗ ∗ ∗
$\mathfrak{M}_{444}(2,2,0)$	D_{12}		$\mathfrak{M}_{444}(2,4,0)$	D_8
$\mathfrak{M}_{444}(2,2,4:1)$	S_3		$\mathfrak{M}_{444}(2,2,4:2)$	2^2
$\mathfrak{M}_{444}(2,2,4:3)$	1		$\mathfrak{M}_{444}(4,2,4:1)$	2^2
$\mathfrak{M}_{444}(4,2,4:2)$	2		$\mathfrak{M}_{444}(4,4,4)$	2
$\mathfrak{M}_{444}(2,2,2)$	2			
$\mathfrak{M}_{445}(0,6,2)$	D_{12}		$\mathfrak{M}_{445}(4,6,2)$	D_8
$\mathfrak{M}_{445}(2,2,2)$	D_8		$\mathfrak{M}_{445}(0,6,4)$	D_8
$\mathfrak{M}_{445}(2,6,4:1)$	S_3		$\mathfrak{M}_{445}(2,6,4:2)$	1
$\mathfrak{M}_{445}(4,2,2:1)$	S_3		$\mathfrak{M}_{445}(4,2,2:2)$	2^2
$\mathfrak{M}_{445}(4,6,4:1)$	2^2		$\mathfrak{M}_{445}(4,6,4:2)$	1
$\mathfrak{M}_{445}(4,6,6:1)$	2^2		$\mathfrak{M}_{445}(4,6,6:2)$	2
$\mathfrak{M}_{445}(4,4,2:1)$	2^2		$\mathfrak{M}_{445}(4,4,2:2)$	3
$\mathfrak{M}_{445}(4,4,2:3)$	2		$\mathfrak{M}_{445}(2,6,6)$	2^2
$\mathfrak{M}_{445}(0,4,4)$	3		$\mathfrak{M}_{445}(2,4,2)$	2
$\mathfrak{M}_{445}(4,4,4:1)$	2		$\mathfrak{M}_{445}(4,4,4:2)$	1
$\mathfrak{M}_{445}(2,6,2)$	2		$\mathfrak{M}_{445}(2,4,4:1)$	2
$\mathfrak{M}_{445}(2,4,4:2)$	1			
$\mathfrak{M}_{446}(0,1,1)$	$2^3.S_4$		$\mathfrak{M}_{446}(2,4,1)$	$3^2.2^2$
$\mathfrak{M}_{446}(0,4,2)$	S_4		$\mathfrak{M}_{446}(4,4,4)$	S_4
$\mathfrak{M}_{446}(2,1,2)$	D_{12}		$\mathfrak{M}_{446}(2,1,5)$	D_8
$\mathfrak{M}_{446}(4,1,2)$	2^3		$\mathfrak{M}_{446}(0,5,2)$	2^3
$\mathfrak{M}_{446}(4,3,1)$	S_3		$\mathfrak{M}_{446}(2,4,3:1)$	S_3
$\mathfrak{M}_{446}(2,4,3:2)$	4		$\mathfrak{M}_{446}(2,3,1)$	2^2
$\mathfrak{M}_{446}(2,4,2)$	2^2		$\mathfrak{M}_{446}(2,5,2:1)$	2^2
$\mathfrak{M}_{446}(2,5,2:2)$	2		$\mathfrak{M}_{446}(4,4,3)$	2^2
$\mathfrak{M}_{446}(4,4,5)$	2^2		$\mathfrak{M}_{446}(4,3,5:1)$	3
$\mathfrak{M}_{446}(4,3,5:2)$	2		$\mathfrak{M}_{446}(4,3,5:3)$	1
$\mathfrak{M}_{446}(0,3,3)$	3		$\mathfrak{M}_{446}(2,2,2)$	2
$\mathfrak{M}_{446}(4,3,2:1)$	2		$\mathfrak{M}_{446}(4,3,2:2)$	1
$\mathfrak{M}_{446}(4,2,2)$	2		$\mathfrak{M}_{446}(4,5,2)$	2
$\mathfrak{M}_{446}(4,5,5)$	2		$\mathfrak{M}_{446}(2,5,5)$	2
$\mathfrak{M}_{446}(2,3,2:1)$	2		$\mathfrak{M}_{446}(2,3,2:2)$	1
$\mathfrak{M}_{446}(2,3,3)$	1		$\mathfrak{M}_{446}(2,3,5)$	1
$\mathfrak{M}_{446}(4,3,3)$	1			
$\mathfrak{M}_{455}(6,6,4)$	D_{12}		$\mathfrak{M}_{455}(6,2,8)$	D_8
$\mathfrak{M}_{455}(2,6,4:1)$	D_8		$\mathfrak{M}_{455}(2,6,4:2)$	2
$\mathfrak{M}_{455}(6,6,8:1)$	S_3		$\mathfrak{M}_{455}(6,6,8:2)$	2^2
$\mathfrak{M}_{455}(6,6,8:3)$	1		$\mathfrak{M}_{455}(4,2,4:1)$	S_3
$\mathfrak{M}_{455}(2,4,4:2)$	2		$\mathfrak{M}_{455}(2,2,8:1)$	2^2

$\mathfrak{M}_{455}(2,2,8:2)$	2		$\mathfrak{M}_{455}(4,4,8:1)$	2
$\mathfrak{M}_{455}(4,4,8:2)$	1		$\mathfrak{M}_{455}(6,2,6)$	2
$\mathfrak{M}_{455}(6,6,6)$	2		$\mathfrak{M}_{455}(2,2,6)$	2
$\mathfrak{M}_{455}(4,6,4)$	2		$\mathfrak{M}_{455}(4,2,8)$	2
$\mathfrak{M}_{455}(4,4,4:1)$	2		$\mathfrak{M}_{455}(4,4,4:2)$	1
$\mathfrak{M}_{455}(6,4,8)$	1		$\mathfrak{M}_{455}(4,6,6)$	1
$\mathfrak{M}_{455}(2,4,6)$	1		$\mathfrak{M}_{455}(4,4,6)$	1

$\mathfrak{M}_{456}(2,1,1)$	D_{12}		$\mathfrak{M}_{456}(6,1,5)$	D_{12}
$\mathfrak{M}_{456}(6,1,4)$	D_8		$\mathfrak{M}_{456}(2,1,4)$	D_8
$\mathfrak{M}_{456}(2,5,5:1)$	D_8		$\mathfrak{M}_{456}(4,1,5)$	D_8
$\mathfrak{M}_{456}(4,4,1)$	S_3		$\mathfrak{M}_{456}(4,1,1)$	S_3
$\mathfrak{M}_{456}(2,2,1:1)$	S_3		$\mathfrak{M}_{456}(2,2,1:2)$	2^2
$\mathfrak{M}_{456}(2,2,1:3)$	2		$\mathfrak{M}_{456}(6,4,4)$	S_3
$\mathfrak{M}_{456}(2,4,5)$	S_3		$\mathfrak{M}_{456}(4,4,5)$	S_3
$\mathfrak{M}_{456}(6,4,1)$	2^2		$\mathfrak{M}_{456}(2,3,1:1)$	2^2
$\mathfrak{M}_{456}(2,3,1:2)$	2		$\mathfrak{M}_{456}(6,2,1)$	2^2
$\mathfrak{M}_{456}(6,5,1:1)$	2^2		$\mathfrak{M}_{456}(6,5,1:2)$	2
$\mathfrak{M}_{456}(4,2,1:1)$	2^2		$\mathfrak{M}_{456}(4,2,1:2)$	1
$\mathfrak{M}_{456}(6,2,2:1)$	2^2		$\mathfrak{M}_{456}(6,2,2:2)$	2
$\mathfrak{M}_{456}(6,2,2:3)$	1		$\mathfrak{M}_{456}(2,2,2:1)$	2^2
$\mathfrak{M}_{456}(2,2,2:2)$	2		$\mathfrak{M}_{456}(2,4,2)$	2^2
$\mathfrak{M}_{456}(6,1,2)$	2^2		$\mathfrak{M}_{456}(6,4,3)$	4
$\mathfrak{M}_{456}(4,4,3:1)$	4		$\mathfrak{M}_{456}(4,4,3:2)$	2
$\mathfrak{M}_{456}(2,2,4)$	2^2		$\mathfrak{M}_{456}(6,2,4)$	2^2
$\mathfrak{M}_{456}(4,3,1:1)$	2		$\mathfrak{M}_{456}(4,3,1:2)$	1
$\mathfrak{M}_{456}(2,3,2:1)$	2		$\mathfrak{M}_{456}(2,3,2:2)$	1
$\mathfrak{M}_{456}(4,1,2)$	2		$\mathfrak{M}_{456}(2,3,3)$	2
$\mathfrak{M}_{456}(4,1,3)$	2		$\mathfrak{M}_{456}(6,2,3)$	2
$\mathfrak{M}_{456}(2,2,3)$	2		$\mathfrak{M}_{456}(2,5,3)$	2
$\mathfrak{M}_{456}(4,5,3)$	2		$\mathfrak{M}_{456}(6,3,3:1)$	2
$\mathfrak{M}_{456}(6,3,3:2)$	1		$\mathfrak{M}_{456}(6,5,4)$	2
$\mathfrak{M}_{456}(2,5,4)$	2		$\mathfrak{M}_{456}(4,4,4)$	2
$\mathfrak{M}_{456}(2,3,5)$	2		$\mathfrak{M}_{456}(6,2,3)$	2
$\mathfrak{M}_{456}(4,5,5:1)$	2		$\mathfrak{M}_{456}(4,5,5:2)$	1
$\mathfrak{M}_{456}(6,3,1)$	2			
$\mathfrak{M}_{456}(4,5,1:1)$	2		$\mathfrak{M}_{456}(4,5,1:2)$	1
$\mathfrak{M}_{456}(4,4,2)$	2		$\mathfrak{M}_{456}(2,5,5:2)$	2
$\mathfrak{M}_{456}(4,2,5)$	2		$\mathfrak{M}_{456}(6,2,5)$	2
$\mathfrak{M}_{456}(4,2,2)$	1		$\mathfrak{M}_{456}(4,5,2)$	1
$\mathfrak{M}_{456}(6,5,2)$	1		$\mathfrak{M}_{456}(4,3,2)$	1
$\mathfrak{M}_{456}(6,3,2)$	1		$\mathfrak{M}_{456}(4,2,3)$	1
$\mathfrak{M}_{456}(6,3,4)$	1		$\mathfrak{M}_{456}(4,3,4)$	1

$\mathfrak{M}_{456}(4,2,4)$	1		$\mathfrak{M}_{456}(4,5,4)$	1
$\mathfrak{M}_{456}(4,3,5)$	1			
$\mathfrak{M}_{466}(1,4,5)$	$(S_3 \times S_4)^+$		$\mathfrak{M}_{466}(1,1,3)$	$D_8.2^2$
$\mathfrak{M}_{466}(4,4,4)$	S_4		$\mathfrak{M}_{466}(4,5,3)$	D_8
$\mathfrak{M}_{466}(2,4,3)$	D_8		$\mathfrak{M}_{466}(5,5,5)$	2^3
$\mathfrak{M}_{466}(1,3,2)$	S_3		$\mathfrak{M}_{466}(2,5,5)$	S_3
$\mathfrak{M}_{466}(4,2,2)$	S_3		$\mathfrak{M}_{466}(2,2,3)$	2^2
$\mathfrak{M}_{466}(2,1,1)$	2^2		$\mathfrak{M}_{466}(4,5,1)$	2^2
$\mathfrak{M}_{466}(3,4,2)$	2^2		$\mathfrak{M}_{466}(2,2,2)$	2^2
$\mathfrak{M}_{466}(3,5,4)$	3		$\mathfrak{M}_{466}(2,4,1)$	2
$\mathfrak{M}_{466}(2,2,1)$	2		$\mathfrak{M}_{466}(3,4,1)$	2
$\mathfrak{M}_{466}(1,3,1)$	2		$\mathfrak{M}_{466}(3,5,2:1)$	2
$\mathfrak{M}_{466}(3,5,2:2)$	1		$\mathfrak{M}_{466}(3,3,2:1)$	2
$\mathfrak{M}_{466}(3,3,2:2)$	1		$\mathfrak{M}_{466}(3,2,2:1)$	2
$\mathfrak{M}_{466}(3,2,2:2)$	1		$\mathfrak{M}_{466}(2,5,2)$	2
$\mathfrak{M}_{466}(5,5,2)$	2		$\mathfrak{M}_{466}(2,2,4)$	2
$\mathfrak{M}_{466}(3,3,5)$	2		$\mathfrak{M}_{466}(3,3,3)$	1
$\mathfrak{M}_{466}(3,2,3)$	1		$\mathfrak{M}_{466}(2,3,1)$	1
$\mathfrak{M}_{466}(5,5,1)$	1		$\mathfrak{M}_{466}(5,3,1)$	1
$\mathfrak{M}_{466}(5,2,1)$	1			
*************	*****		*************	*****
$\mathfrak{M}_{555}(4,4,8:1)$	2^2		$\mathfrak{M}_{555}(4,4,8:2)$	2
$\mathfrak{M}_{555}(4,4,8:3)$	1		$\mathfrak{M}_{555}(8,4,8)$	3
$\mathfrak{M}_{555}(8,8,8:1)$	2		$\mathfrak{M}_{555}(8,8,8:2)$	1
$\mathfrak{M}_{555}(4,4,6)$	2		$\mathfrak{M}_{555}(6,4,8)$	1
$\mathfrak{M}_{555}(8,6,8)$	1		$\mathfrak{M}_{555}(6,6,6)$	1
$\mathfrak{M}_{555}(6,4,6)$	1			
$\mathfrak{M}_{556}(4,5,1:1)$	S_3		$\mathfrak{M}_{556}(4,5,1:2)$	2
$\mathfrak{M}_{556}(8,5,1)$	S_3		$\mathfrak{M}_{556}(4,5,4)$	2^2
$\mathfrak{M}_{556}(6,1,1)$	2		$\mathfrak{M}_{556}(6,2,1:1)$	2
$\mathfrak{M}_{556}(6,2,1:2)$	1		$\mathfrak{M}_{556}(8,1,1:1)$	2
$\mathfrak{M}_{556}(8,1,1:2)$	1		$\mathfrak{M}_{556}(8,4,1:1)$	2
$\mathfrak{M}_{556}(8,4,1:2)$	1		$\mathfrak{M}_{556}(8,3,1:1)$	2
$\mathfrak{M}_{556}(8,3,1:2)$	1		$\mathfrak{M}_{556}(4,4,1)$	2
$\mathfrak{M}_{556}(8,2,1:1)$	2		$\mathfrak{M}_{556}(8,2,1:2)$	1
$\mathfrak{M}_{556}(4,3,1)$	2		$\mathfrak{M}_{556}(6,5,1)$	2
$\mathfrak{M}_{556}(4,2,1:1)$	2		$\mathfrak{M}_{556}(4,2,1:2)$	1
$\mathfrak{M}_{556}(6,5,2:1)$	2		$\mathfrak{M}_{556}(6,5,2:2)$	1
$\mathfrak{M}_{556}(4,2,2:1)$	2		$\mathfrak{M}_{556}(4,2,2:2)$	1
$\mathfrak{M}_{556}(8,5,2:1)$	2		$\mathfrak{M}_{556}(8,5,2:2)$	1
$\mathfrak{M}_{556}(4,3,2)$	2		$\mathfrak{M}_{556}(4,3,3:1)$	2
$\mathfrak{M}_{556}(4,3,3:2)$	1		$\mathfrak{M}_{556}(6,5,5)$	2

$\mathfrak{M}_{556}(8,5,5)$	2		$\mathfrak{M}_{556}(6,3,1)$	1
$\mathfrak{M}_{556}(6,4,1)$	1		$\mathfrak{M}_{556}(8,4,2)$	1
$\mathfrak{M}_{556}(6,3,2)$	1		$\mathfrak{M}_{556}(8,3,2)$	1
$\mathfrak{M}_{556}(8,2,2)$	1		$\mathfrak{M}_{556}(6,4,2)$	1
$\mathfrak{M}_{556}(4,4,2)$	1		$\mathfrak{M}_{556}(6,4,3)$	1
$\mathfrak{M}_{556}(6,5,3)$	1		$\mathfrak{M}_{556}(8,5,3)$	1
$\mathfrak{M}_{556}(4,5,3)$	1		$\mathfrak{M}_{556}(4,4,3)$	1
$\mathfrak{M}_{556}(6,4,4)$	1		$\mathfrak{M}_{556}(6,5,4)$	1
$\mathfrak{M}_{557}(6,1,1)$	1			
$\mathfrak{M}_{566}(5,5,2)$	S_3		$\mathfrak{M}_{566}(2,5,5)$	S_3
			$\mathfrak{M}_{566}(5,5,3)$	2^2
$\mathfrak{M}_{566}(1,1,2)$	2^2		$\mathfrak{M}_{566}(1,1,4:1)$	2^2
$\mathfrak{M}_{566}(1,1,4:2)$	2		$\mathfrak{M}_{566}(1,2,5)$	2^2
$\mathfrak{M}_{566}(4,2,5)$	2^2		$\mathfrak{M}_{566}(3,4,5)$	4
$\mathfrak{M}_{566}(3,3,5)$	4		$\mathfrak{M}_{566}(3,4,3)$	2
$\mathfrak{M}_{566}(3,3,3)$	2		$\mathfrak{M}_{566}(4,4,3)$	2
$\mathfrak{M}_{566}(1,1,3)$	2		$\mathfrak{M}_{566}(1,1,1)$	2
$\mathfrak{M}_{566}(1,4,1)$	2		$\mathfrak{M}_{566}(3,2,1:1)$	2
$\mathfrak{M}_{566}(3,2,1:2)$	1		$\mathfrak{M}_{566}(5,5,1)$	2
$\mathfrak{M}_{566}(1,5,2)$	2		$\mathfrak{M}_{566}(4,5,2)$	2
$\mathfrak{M}_{566}(1,2,2)$	2		$\mathfrak{M}_{566}(2,5,2)$	2
$\mathfrak{M}_{566}(2,2,2:1)$	2		$\mathfrak{M}_{566}(2,2,2:2)$	1
$\mathfrak{M}_{566}(4,3,2)$	2		$\mathfrak{M}_{566}(5,2,2)$	2
$\mathfrak{M}_{566}(4,4,2:1)$	2		$\mathfrak{M}_{566}(4,4,2:2)$	1
$\mathfrak{M}_{566}(1,4,2)$	2		$\mathfrak{M}_{566}(1,3,3)$	2
$\mathfrak{M}_{566}(3,2,4)$	2		$\mathfrak{M}_{566}(5,5,4)$	2
$\mathfrak{M}_{566}(2,4,4)$	2		$\mathfrak{M}_{566}(3,2,5)$	2
$\mathfrak{M}_{566}(2,2,5)$	2		$\mathfrak{M}_{566}(5,1,1)$	1
$\mathfrak{M}_{566}(2,5,3)$	1		$\mathfrak{M}_{566}(2,2,3)$	1
$\mathfrak{M}_{566}(1,2,1)$	1		$\mathfrak{M}_{566}(3,4,1)$	1
$\mathfrak{M}_{566}(4,2,1)$	1		$\mathfrak{M}_{566}(1,3,2)$	1
$\mathfrak{M}_{566}(3,3,1)$	1		$\mathfrak{M}_{566}(1,3,1)$	1
$\mathfrak{M}_{566}(4,5,1)$	1		$\mathfrak{M}_{566}(3,5,1)$	1
$\mathfrak{M}_{566}(5,2,1)$	1		$\mathfrak{M}_{566}(4,4,1)$	1
$\mathfrak{M}_{566}(3,3,2)$	1		$\mathfrak{M}_{566}(4,2,2)$	1
$\mathfrak{M}_{566}(2,3,2)$	1			
* * * * * * * * * * * **	* * * * *		* * * * * * * * * * * **	* * * * *
$\mathfrak{M}_{666}(5,5,2)$	$3^2.2$		$\mathfrak{M}_{666}(5,1,3)$	2^2
$\mathfrak{M}_{666}(3,2,3)$	2^2		$\mathfrak{M}_{666}(5,1,2)$	2^2
$\mathfrak{M}_{666}(4,1,4)$	3		$\mathfrak{M}_{666}(2,1,3)$	2
$\mathfrak{M}_{666}(5,1,1)$	2		$\mathfrak{M}_{666}(2,2,1:1)$	2
$\mathfrak{M}_{666}(1,2,2:2)$	1		$\mathfrak{M}_{666}(4,1,2)$	2

$\mathfrak{M}_{666}(2,1,1)$	1		$\mathfrak{M}_{666}(1,4,1)$	1

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