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INVESTIGATION OF SOME EVEN-EVEN Mo NUCLEI IN RELATIVISTIC MEAN FIELD THEORY

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Abstract. - The structures of the nuclei on isotope chain of even-even Mo are investigated in the axially deformed relativistic mean-field theory with the NL-SH forces. We put an emphasis on the ground state properties of molybdenum nuclei. With high neutron number is correctly reproduced in the relativistic mean-field theory (RMF). In general, the RMF theory can give a good description of the isotope chain of Mo nuclei.

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Relativistic mean field, even-even Mo nuclei, binding energy, deformations

One of the main aims of researches in nuclear physics is to try to describe ground-state properties of nuclei in the whole mass region. Unfortunately, due to lack of understanding in strong interaction and numerical difficulty in treating nuclear many-body problems, so far all microscopic descriptions are only possible on a phenomenological ground. The relativistic mean field theory of the nucleus has been a fairly successful application of Dirac phenomenology. It has been applied to study the binding energy, the shape of the ground state and various other properties of nuclei. An excellent recent review by Ring [1], gives a comprehensive list of references. Recent advances include studies in dripline regions through the relativistic Hartree-Bogoliubov formalism [2], explanation of pseudospin symmetry [3], etc.

The basic Ansatz of the RMF theory is a Lagrangian density [1, 4-6] where nucle-

ons are described as Dirac particles which interact via the exchange of various mesons. The Lagrangian density considered is written in the form:

$$\begin{aligned} L = \bar{\psi}(i \not{\partial} - M)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma \ - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} \\ + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\ \vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \\ \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\ \not{\omega}\psi - g_\rho\bar{\psi}\ \not{\rho}\vec{\tau}\psi \ - e\bar{\psi}\ \not{A}\psi \end{aligned} \tag{1}$$

The meson fields included are the isoscalar σ-meson, the isoscalar-vector ω-meson and the isovector-vector ρ-meson. The latter provides the necessary isospin asymmetry. The arrows in Eq. (1) denote the isovector quantities. The Lagrangian contains also a non-linear scalar self-interaction of the σ-meson.

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 \tag{2}$$

This term is important for appropriate description of surface properties [7]. M, m_σ,m_ω and m_ρ are the nucleon-, the σ-, the ω- and the ρ-meson masses respectively, while g_σ, g_ω, g_ρ and $\frac{e^2}{4\pi} = \frac{1}{137}$ are the corresponding coupling constants for the mesons and the photon. The field tensors of the vector mesons and of the electromagnetic fields take the following form:

$$\Omega^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu \tag{3}$$

$$\vec{R}^{\mu\nu} = \partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu \tag{4}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{5}$$

The variational principle gives the equations of motion. The mean field approximation is introduced at this stage by treating the fields as the c-number or classical fields. This results into a set of coupled equations namely the Dirac equation with potential terms for the nucleons and the Klein-Gordon type equations with sources for the mesons and the photon. For the static case, along with the time reversal invariance and charge conservation the equations get simplified. The resulting equations, known as RMF equations have the following form.

Dirac equation for the nucleon,

$$\{-i\alpha \bigtriangledown + V(r) + \beta\,[M + S(r)]\}\psi_i = \varepsilon_i\psi_i \tag{6}$$

where $V(r)$ represents the vector potential:

$$V(r) = g_\omega \omega_0(r) + g_\rho \tau_3 \rho_0(r) + e\frac{1+\tau_3}{2} A_0(r) \tag{7}$$

and $S(r)$ is the scalar potential:

$$S(r) = g_\sigma \sigma(r) \tag{8}$$

the latter contributes to the effective mass as:

$$M^* = M + S(r) \tag{9}$$

The Klein-Gordon equations for the meson and electromagnetic fields with the nucleon densities as sources:

$$\{-\Delta + m_\sigma^2\}\sigma(r) = -g_\sigma \rho_s(r) - g_2\sigma^2(r) - g_3\sigma^3(r) \tag{10}$$

$$\{-\Delta + m_\omega^2\}\omega_0(r) = g_\omega \rho_\nu(r) \tag{11}$$

$$\{-\Delta + m_\rho^2\}\rho_0(r) = g_\rho \rho_3(r) \tag{12}$$

$$-\Delta A_0(r) = e\rho_c(r) \tag{13}$$

The corresponding densities are:

$$\rho_s = \sum_{i=1}^{A} n_i \bar{\psi}_i \psi_i \tag{14}$$

$$\rho_\nu = \sum_{i=1}^{A} n_i \psi_i^\dagger \psi_i \tag{15}$$

$$\rho_3 = \sum_{i=1}^{Z} n_i \psi_p^\dagger \psi_p - \sum_{i=1}^{N} n_i \psi_n^\dagger \psi_n \tag{16}$$

$$\rho_c = \sum_{p=1}^{Z} n_i \psi_p^\dagger \psi_p \tag{17}$$

Here the sums are taken over the particle states only. This implies that the contributions from negative-energy states are neglected (no-sea approximation), i.e. the vacuum is not polarized. The π-meson does not contribute in the present relativistic mean field (Hartree) approximation because of its pseudo nature. The occupation number n_i is introduced to account for pairing which is important for open shell nuclei. In the absence of pairing it takes the value one (zero) for the levels below (above) the Fermi surface. In the presence of pairing the partial occupancies (n_i) are obtained in the constant gap approximation (BCS) through the well known expression:

$$n_i = \frac{1}{2}(1 - \frac{\varepsilon_i - \lambda}{\sqrt{(\varepsilon_i - \lambda)^2 + \Delta^2}}) \tag{18}$$

The ε_i is the single-particle energy for the state i and chemical potential or Fermi energy λ for protons (neutrons) is obtained from the requirement

$$\sum_{i=1} n_i = the\ number\ of\ protons(Z)$$
$$(the\ number\ of\ neutrons(N)) \tag{19}$$

The sum is taken over protons (neutrons) states. The gap parameter Δ is calculated from the observed odd-even mass differences. In the absence of experimental masses it can inferred from the extrapolation of the masses given by any of the avalaible mass formulae.

The above set of equations (6,10,11,12 and 13) are to be solved self-consistently. For this purpose one starts with an initial guess of the fields (e.g. generated by axially deformed Woods-Saxon potential) to calculate the potential terms (Eqs. (7, 8)) appearing in the Dirac equation (Eq. (6)). The Dirac equation is solved with these potentials terms to yield the nucleon spinors which in term are used to obtain the sources (densities). The meson and photon equations are then solved with these sources to get a new set of fields to be used for the calculation of new potential terms. The Dirac equation is then solved with the new potentials to get the spinors again to be used to obtain the new sources for the meson fields. This iterative procedure is continued till the converegence upto the desired accuracy is achieved.
The total binding energy is written as

$$E = E_{part} + E_\sigma + E_\omega + E\rho + E_{Coul} + E_{pair} + E_{nl} - E_{c.m} - AM. \tag{20}$$

DETAILS OF CALCULATIONS

The molybdenum nuclei considered here are even-mass nuclei with mass number $A = 88$ up to 106. All of these isotopes are open-shell nuclei both in protons and neutrons, thus requiring the inclusion of pairing. The parameter set NL-SH [8] has been employed for all nuclei. This set has been found to be very successful for the ground-state properties of many nuclei. The number of shells taken into account are 12 and 20 for the fermionic (N_F) and bosonic (N_B) expansion, respectively. The basis parameters $\hbar\omega$ and β_0 used for the calculations have been taken to be $41A^{-1/3}$ and 0.0, respectively.

In order to investigate these Mo nuclei we have performed the calculations with Saxoon-Woods initial wavefunctions.

There are a number of parametrization sets for prediction of the nuclear ground state properties [1]. We used the parameter set NL-SH [8] in the present calculation.

Table 1: The parameters of the force NL-SH. All the masses are in MeV, while g_2 is in fm^{-1}. The other coupling constants are dimensionless.

$M = 939.0$	$m_\sigma = 526.059$	$m_\omega = 783.0$	$m_\rho = 763.0$
	$g_\sigma = 10.444$	$g_\omega = 12.945$	$g_\rho = 4.383$
	$g_2 = -6.9099$	$g_3 = -15.8337$	

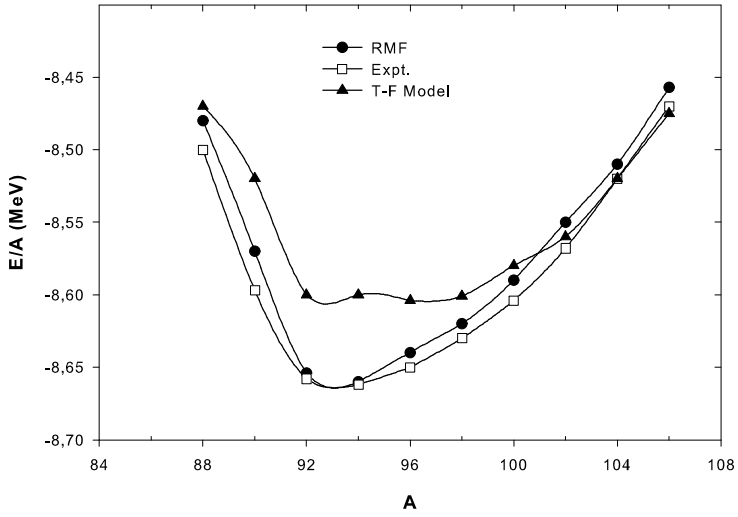

Figure 1: The calculated binding energy per nucleon for Mo isotopes.

Fig.1 shows the binding energy per nucleon (E/A) for Mo isotopes. The empirical values which is from Thomas Fermi Model taken from [9] are also shown. The parabolic shapes of the binding energy per nucleon emerges nicely. The minimum in the binding energy is observed at the magic neutron number N = 50 in RMF theory. The calculated RMF binding energies agree very closely the experimental values [10].

We give in Fig.2 the quadrupole deformation β_2 for the shape correspondingto the lowest energy. It is seen that β_2 is close to zero in the ligher isotopes of molybdenum. This turns out that these nuclei are spherical near the magic neutron number N = 50. Nuclei above this magic number the shape turns to prolate one. In Fig.3 the r.m.s. charge and neutron radii of Mo nuclei are shown. It is seen that ongoing from the lighter isotopes to the heavier ones the charge radii exhibit a decreasing trend upto the magic isotope, that is the lighter isotopes have higher charge radii than the heavier closed neutron- shell nucleus. The charge radii for nuclei heavier than the closed neutron-shell start increasing with addition of neutrons. The neutron

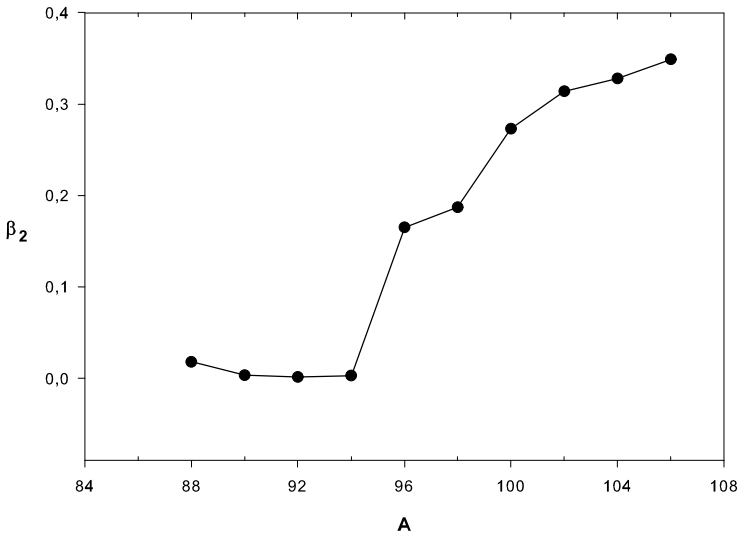

Figure 2: The quadrupole deformation β_2 for Mo nuclei.

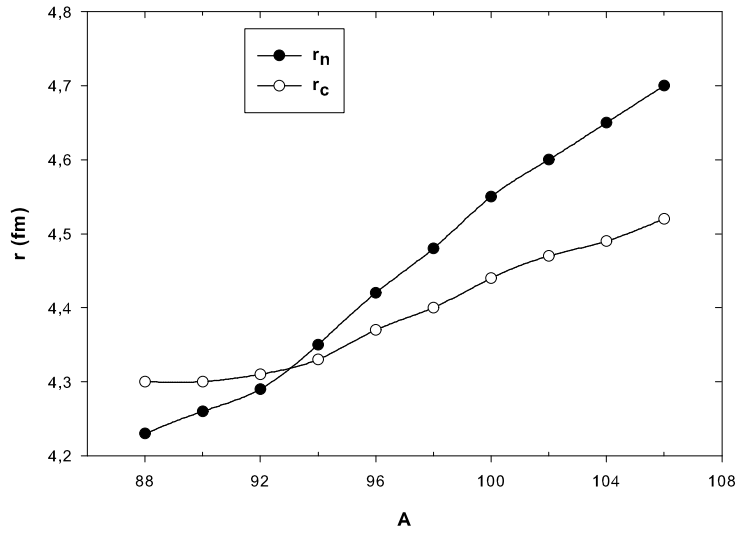

Figure 3: The calculated r.m.s. charge and neutron radii of Mo nuclei.

Table 2: Some calculated energy values of Mo isotopic chain obtained by using the force NL-SH.

$E(MeV)$	E_σ	$E_{\sigma nl}$	E_ω	E_ρ	E_C	E_{pair}	E_{cm}	E_{total}
^{88}Mo	-12493.1	-236.88	10526.7	2.59	281.84	-12.63	-6.91	-744.06
^{90}Mo	-12897.9	-244.16	10869.5	5.02	281.53	-12.32	-6.86	-770.72
^{92}Mo	-13289.6	-251.12	11201.18	8.26	281.09	-11.39	-6.81	-796.16
^{94}Mo	-13489.6	-255.87	11365.02	11.79	279.68	-13.31	-6.76	-810.37
^{96}Mo	-13763.9	-261.53	11591.1	16.13	277.50	-12.48	-6.72	-827.34
^{98}Mo	-13969.7	-266.17	11759.3	20.87	275.99	-12.74	-6.67	-841.91
^{100}Mo	-14256.3	-271.56	11995.9	26.39	274.19	-12.48	-6.62	-857.76
^{102}Mo	-14536.5	-277.06	12225.7	32.57	272.82	-12.55	-6.58	-871.25
^{104}Mo	-14783.2	-282.20	12427.8	39.15	271.52	-12.75	-6.54	-884.11
^{106}Mo	-15017.09	-287.07	12619.2	46.06	270.18	-12.82	-6.49	-896.49

Table 3: The calculated quadrupole values of Mo nuclei.

	$Q_n(b)$	$Q_p(b)$	$Q(b)$
^{88}Mo	0.18	0.15	0.33
^{90}Mo	0.032	0.031	0.063
^{92}Mo	0.011	0.013	0.02
^{94}Mo	0.032	0.027	0.059
^{96}Mo	2.022	1.597	3.619
^{98}Mo	2.467	1.775	4.242
^{100}Mo	3.761	2.665	6.425
^{102}Mo	4.577	3.051	7.628
^{104}Mo	5.009	3.212	8.221
^{106}Mo	5.559	3.477	9.036

radii, on the other hand, also show a kink about the neutron shell closure. However, the neutron radius for lighter isotopes in these chains is not higher than that of the closed-shell nucleus.

In Table 2 we present some calculated energy values of molybdenum nuclei. It is also given calculated quadrupole moments of neutron and proton in Mo isotopes in Table 3.

SUMMARY

The relativistic mean field theory has been used to study a number of Mo isotopes in the mass 88-106 region. The ground state properties of these nuclei have been calculated using rmfaxial code [11]. It is found out that the rmf theory is capable of describing Mo nuclei.

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