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Some rank 3 residually connected geometries for M_{23}

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Abstract. In this paper we shall give more information about some rank 3 residually connected geometries for M_{23} and describe them more clearly. In [10], Kilic calculated all rank 3 residually connected geometries for M_{23}. So this paper is based on the idea of [10].

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Keywords: Mathieu groups, Steiner system, group geometries

1. Introduction and Notation

We begin by reviewing geometries and some standard notation. Let I, Γ be sets, where I is a finite and let t be map from Γ to I. Then the triple $(\Gamma, *, t)$, where $*$ is a symmetric incidence relation on Γ, is a geometry provided that whenever $x * y$ $(x, y \in \Gamma)$ then $t(x) \neq t(y)$. It is usual we will do it here, to write Γ instead of $(\Gamma, *, t)$ and to say Γ is a geometry. The map $t : \Gamma \to I$ is called the type map and we say $x \in \Gamma$ has type i if $t(x) = i$. Also for $x, y \in \Gamma$ if $x * y$, then we will say x and y are incident. The rank of the geometry is the cardinality of $t(\Gamma)$. For $i \in I$, $\Gamma_i = \{x \in \Gamma \mid t(x) = i\}$; so Γ_i consist of all elements of Γ which have type i. Suppose Γ is a geometry, for $x \in \Gamma$, the residue of x is $\Gamma_x = \{y \in \Gamma \mid x * y\}$. The notation of the residue is important in the theory of geometries note that $(\Gamma_x, * \mid \Gamma_x, t)$ is a geometry in its own right (where $* \mid \Gamma_x$ is the restriction of $*$ to Γ_x). Also we note that for every $y \in \Gamma_x$, $t(x) \neq t(y)$. A flag F of Γ is a subset of Γ which, for all $x, y \in F$, $x \neq y$, $x * y$. Let Γ be a geometry and F a flag of Γ. The type of F is the subset $t(F)$ of I and the rank (respectively corank) of F is the cardinality of $t(F)$ (respectively $I \setminus t(F)$). A chamber of Γ is flag of rank I. All geometries we consider are assumed to contain at least one flag of rank $\mid I \mid$. The automorphism group of Γ, $Aut\Gamma$, consist of all permutations of Γ which preserve the sets Γ_i and the incidence relation $*$. Let G be a subgroup of $Aut\Gamma$. We call Γ a flag transitive

geometry for G if for any two flags F_1 and F_2 of Γ having the same type, there exists $g \in G$ such that ${F_1}^g = F_2$.

A geometry Γ is called residually connected if for all flags F of Γ of corank 2 the incidence graph of Γ_F is connected. Now suppose that Γ is a flag transitive geometry for the group G. As is well-known we may view Γ in terms of certain cosets of G. This is the approach we shall follow here. For each $i \in I$ choose an $x_i \in \Gamma_i$ and set $G_i = Stab_G(x_i)$. Let $\mathcal{F}$=$\{G_i : i \in I\}$. We now define a geometry $\Gamma(G, \mathcal{F})$ where the objects of type i in $\Gamma(G, \mathcal{F})$ are the right cosets of G_i in G and for $G_i x$ and $G_j y$ $(x, y \in G, i, j \in I)$ $G_i x \star G_j y$ whenever $G_i x \cap G_j y \neq \emptyset$. Also by letting G act upon $\Gamma(G, \mathcal{F})$ by right multiplication we see that $\Gamma(G, \mathcal{F})$ is a flag transitive geometry for G. Moreover Γ and $\Gamma(G, \mathcal{F})$ are isomorphic geometries for G. So we shall be studying geometries of the form $\Gamma(G, \mathcal{F})$, where $G \cong M_{23}$ and G_i is a maximal subgroup of G for all $i \in I$.

Rank 2 geometries of M_{23} (the Mathieu Group of degree 23) were investigated in [9]. Kilic, in [10], calculated all rank 3 residually connected geometries for the Mathieu group M_{23} whose object stabilizer are maximal subgroups. Now we give explicit description these geometries and describe them more clearly using the figures. Some of the geometries very similar to each other. In this paper, we can see the differences of these geometries. We shall use the result of all rank 2 geometries of M_{23} calculated in [9].

For the remainder of this paper G will denote M_{23}, the Mathieu Group of degree 23. Also Ω will denote a 24 element set possessing the Steiner system $S(24, 8, 5)$ as described by Curtis's MOG [4]. We will follow the notation of [4].

So $\Omega =$ $\begin{array}{|c|c|c|}\hline O_1 & O_2 & O_3 \\ \hline \end{array}$ $=$ $\begin{array}{|cc|cc|cc|}\hline \infty & 14 & 17 & 11 & 22 & 19 \\ 23 & 8 & 4 & 13 & 1 & 9 \\ 3 & 20 & 16 & 7 & 12 & 5 \\ 15 & 18 & 10 & 2 & 21 & 6 \\ \hline \end{array}$, where O_1, O_2 and O_3 are the heavy bricks of the MOG. Here M_{24} is the Mathieu group of degree 24 which leaves invariant the Steiner system $S(24, 8, 5)$ on Ω. Set $\Lambda = \Omega \setminus \{\infty\}$

An octad of Ω is just an 8-element block of the Steiner system and a subset of Ω is called a dodecad if it is the symmetric difference of two octads of Ω which intersect in a set of size two. Corresponding to each 4 points of Ω there is a partition of the 24 points into 6 tetrads with the property that the union of any two tetrads is an octad, this configuration will be called a sextet. The following sets will appear when we describe geometries for G.

(i) $\mathcal{D}$=$\{X \subseteq \Lambda || X| = 2\}$ (duads of Λ).
(ii) $\mathcal{H}$=$\{X \subseteq \Lambda | X \cup \{\infty\}$ is an octad of Ω $\}$(heptads of Λ).
(iii) $\mathcal{O}$=$\{X \subseteq \Lambda | X$ is an octad of $\Omega\}$ (octads of Λ).
(iv) $\mathcal{D}_{\mathrm{o}}$=$\{X \subseteq \Lambda | X$ is a dodecad of $\Omega\}$ (dodecads of Λ).
(v) $\mathcal{S}$=$\{X_i \subseteq \Omega || X_i| = 4$ (for each $i \in I$), $X_i \cup X_j$ is an octad $(i \neq j)$ and

$\Omega = \dot{\bigcup}_{i\in I} X_i$, $i \in I = \{1...6\}\}$ (sextets of Ω).

From the [3], the conjugacy classes of the maximal subgroups of G are as follows:

Order	*Index*	M_i	*Description*
443520	23	$M_1 \cong M_{22}$	$M_1 = Stab_G\{a\}$, $a \in \Lambda$
40320	253	$M_2 \cong L_3(4) : 2b$	$M_2 = Stab_G\{X\}$, $X \in \mathcal{D}$
40320	253	$M_3 \cong 2^4 : A_7$	$M_3 = Stab_G\{X\}$, $X \in \mathcal{H}$
20160	506	$M_4 \cong A_8$	$M_4 = Stab_G\{X\}$, $X \in \mathcal{O}$
7920	1288	$M_5 \cong M_{11}$	$M_5 = Stab_G\{X\}$, $X \in \mathcal{D}_o$
5760	1771	$M_6 \cong 2^4 : (3 \times A_5) : 2$	$M_6 = Stab_G\{X\}$, $X \in \mathcal{S}$
253	40320	$M_7 \cong 23 : 11$	

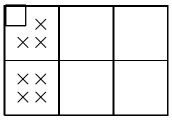
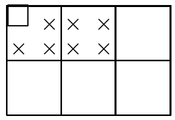
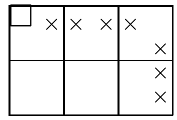
For $i \in \{1, ..., 7\}$, we let $\mathfrak{M}_i$ denote the conjugacy class of M_i, M_i as given in the previous table. We also set $\mathfrak{M}=\bigcup_{i=1}^{7} \mathfrak{M}_i$; so $\mathfrak{M}$ consist of all maximal subgroups of G. In [4] and [8], we can find further information about 23:11. Also put $\mathfrak{X} = \Lambda \cup \mathcal{D} \cup \mathcal{H} \cup \mathcal{O} \cup \mathcal{D}_o \cup \mathcal{S}$.

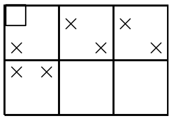
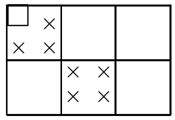
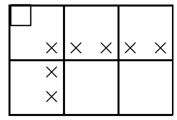
Suppose G_1 and G_2 are maximal subgroups of G with $G_1 \neq G_2$. Set $G_{12} = G_1 \cap G_2$. We use $\mathfrak{M}_{ij}(t)$ to describe $\{G_1, G_2, G_1 \cap G_2\}$ according to the following scheme: $G_1 \in \mathfrak{M}_i$, $G_2 \in \mathfrak{M}_j$ (and so $G_1 = Stab_G(X_1)$ and $G_2 = Stab_G(X_2)$ for some appropriate subsets X_1 and X_2 of Λ in $\mathfrak{X}$) with $|X_1 \cap X_2| = t$. When listing up the rank 2 geometries of G in [9] the notation $\mathfrak{M}_{ij}(t)$ is not sufficient enough to describe the geometries up to conjugacy in $AutG$. All calculations in $2^4 : (3 \times A_5) : 2$ and $23 : 11$ we can not use this notation, we shall use the following notation; $\mathfrak{M}_{46}(1)$ means the first case of the intersection of octad and sextet, $\mathfrak{M}_{46}(2)$ means the second case of the intersection of octad and sextet. In [9], we shall find more information about it.

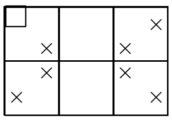
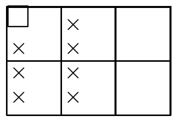
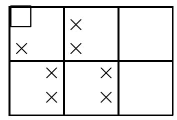
Now suppose we have three distinct maximal subgroups of G; G_1, G_2 and G_3. We shall use use G_{12},G_{13}, G_{23} and G_{123} to denote, respectively $G_1 \cap G_2$, $G_1 \cap G_3$, $G_2 \cap G_3$ and $G_1 \cap G_2 \cap G_3$. We extend the above notation using $\mathfrak{M}_{ijk}(t_{ij}, t_{ik}, t_{jk})$ to indicate that $G_1 \in \mathfrak{M}_i$, $G_2 \in \mathfrak{M}_j$, $G_3 \in \mathfrak{M}_k$ with $|X_i \cap X_j| = t_{ij}$,$|X_i \cap X_k| = t_{ik}$ and $|X_j \cap X_k| = t_{jk}$. (Here $G_1 = Stab_G(X_i)$, $G_2 = Stab_G(X_j)$, $G_3 = Stab_G(X_k)$ for suitable X_i, X_j and X_k of $\Lambda \in \mathfrak{X}$). Again we run into the possibility that in some instances, we need further subdivide these cases, and we do this using the ad hoc notation $\mathfrak{M}_{ijk}(t_{ij}, t_{ik}, t_{jk} : l)$ where $l \in \{1, 2, 3, 4\}$. The aim of this paper is to see the differences of these geometries. We note that if two or more i, j and k are equal, apparently different parameters t_{ij}, t_{ik}, t_{jk} may describe the same situation. For examle $\mathfrak{M}_{344}(2, 0, 4)$ and $\mathfrak{M}_{344}(0, 2, 4)$ describe the same configuration as do $\mathfrak{M}_{333}(3, 1, 1)$ and $\mathfrak{M}_{333}(1, 3, 1)$.

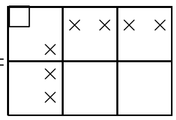
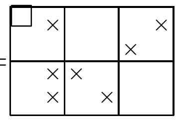
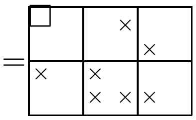
We remark that the geometry $\Gamma(G, \mathcal{F})$ where $\mathcal{F} = \{G_1, G_2, G_3\}$ is residually connected if and only if $G_1 =< G_{12}, G_{13} >$, $G_2 =< G_{12}, G_{23} >$ and $G_3 =< G_{13}, G_{23} >$.

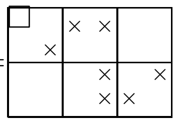
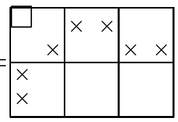
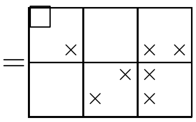
Below we give certain subsets of Λ which will be encountered frequently in our list.

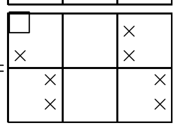
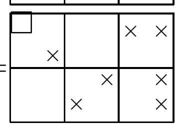
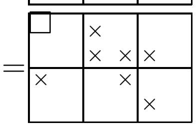
$H_1=$  $H_2=$  $H_3=$ 

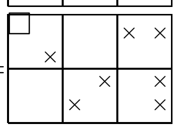
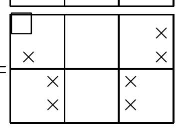
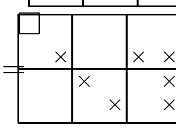
$H_4=$  $H_5=$  $H_6=$ 

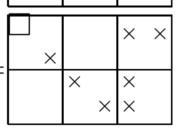
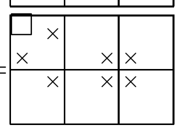
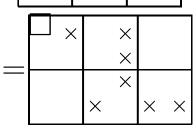
$H_7=$  $H_8=$  $H_9=$ 

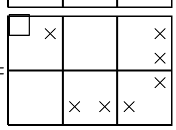
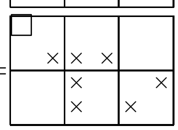
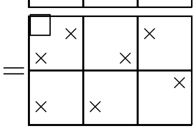
$H_{10}=$  $H_{11}=$  $H_{12}=$ 

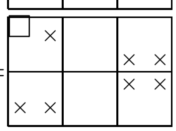
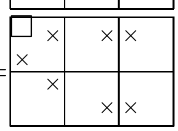
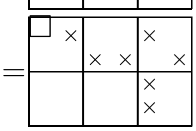
$H_{13}=$  $H_{14}=$  $H_{15}=$ 

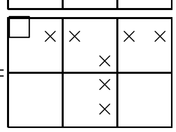
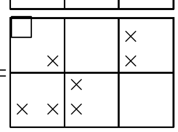
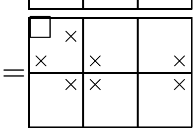
$H_{16}=$  $H_{17}=$  $H_{18}=$ 

$H_{19}=$  $H_{20}=$  $H_{21}=$ 

$H_{22}=$  $H_{23}=$  $H_{24}=$ 

$H_{25}=$  $H_{26}=$  $H_{27}=$ 

$H_{28}=$  $H_{29}=$  $H_{30}=$ 

$H_{31}=$  $H_{32}=$  $H_{33}=$ 

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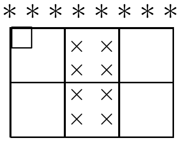
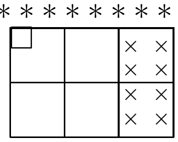
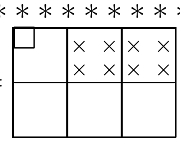
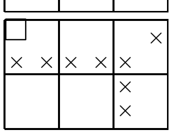
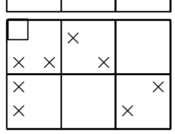
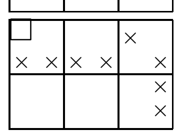
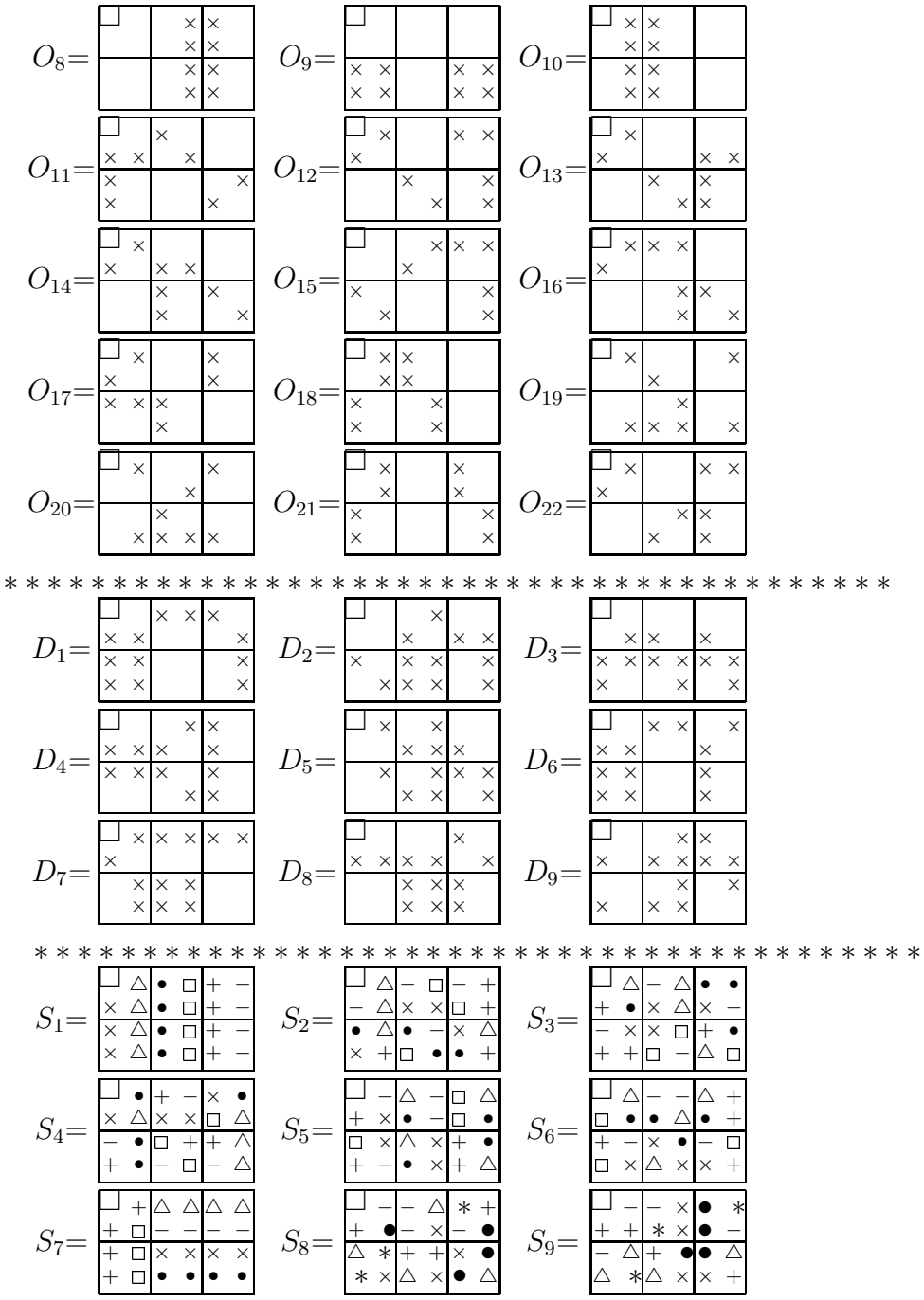
$O_2=$  $O_3=$  $O_4=$ 

$O_5=$  $O_6=$  $O_7=$ 



The notation used in this paper is the same as that used by Kilic in [10].

2. Some rank 3 residually connected geometries for M_{23}

We now define the subgroups for $\mathfrak{M}_{ijk}(t_{ij}, t_{ik}, t_{jk} : l)$ $(l \in \{1, 2, 3, 4\})$ by giving the $X_i \in \mathfrak{X}$ such that $G_i = Stab_G(X_i)$ $(i = 1, 2, 3)$. We are now in a position to give difference of geometries.

$\mathfrak{M}_{\text{ijk}}(t_{ij}, t_{ik}, t_{jk} : l)$	X_1	X_2	X_3
$\mathfrak{M}_{122}(0,0,0:1)$	$\{14\}$	$\{23,8\}$	$\{3,20\}$
$\mathfrak{M}_{122}(0,0,0:2)$	$\{22\}$	$\{15,14\}$	$\{3,20\}$
$\mathfrak{M}_{125}(0,1,2:1)$	$\{14\}$	$\{17,11\}$	D_1
$\mathfrak{M}_{125}(0,1,2:2)$	$\{4\}$	$\{17,11\}$	D_1
$\mathfrak{M}_{125}(0,0,1:1)$	$\{11\}$	$\{14,17\}$	D_1
$\mathfrak{M}_{125}(0,0,1:2)$	$\{3\}$	$\{14,17\}$	D_1
$\mathfrak{M}_{126}(0,1,2:1)$	$\{1\}$	$\{16,19\}$	S_1
$\mathfrak{M}_{126}(0,1,2:2)$	$\{18\}$	$\{14,17\}$	S_1
$\mathfrak{M}_{126}(0,1,2:3)$	$\{13\}$	$\{14,17\}$	S_1
$\mathfrak{M}_{136}(0,1,2:1)$	$\{20\}$	H_2	S_1
$\mathfrak{M}_{136}(0,1,2:2)$	$\{22\}$	H_2	S_1
$\mathfrak{M}_{144}(0,0,4:1)$	$\{23\}$	O_2	O_8
$\mathfrak{M}_{144}(0,0,4:2)$	$\{14\}$	O_2	O_8
$\mathfrak{M}_{145}(0,0,4:1)$	$\{11\}$	O_3	D_1
$\mathfrak{M}_{145}(0,0,4:2)$	$\{23\}$	O_3	D_1
$\mathfrak{M}_{146}(0,1,5:1)$	$\{23\}$	O_2	S_2
$\mathfrak{M}_{146}(0,1,5:2)$	$\{14\}$	O_2	S_2
$\mathfrak{M}_{155}(0,0,8:1)$	$\{17\}$	D_1	D_6
$\mathfrak{M}_{155}(0,0,8:2)$	$\{23\}$	D_1	D_6
$\mathfrak{M}_{156}(1,1,1:1)$	$\{14\}$	D_1	S_1
$\mathfrak{M}_{156}(1,1,1:2)$	$\{4\}$	D_1	S_1
$\mathfrak{M}_{156}(0,1,1:1)$	$\{17\}$	D_1	S_1
$\mathfrak{M}_{156}(0,1,1:2)$	$\{8\}$	D_1	S_1
$\mathfrak{M}_{156}(0,1,2:1)$	$\{18\}$	D_2	S_1
$\mathfrak{M}_{156}(0,1,2:2)$	$\{10\}$	D_2	S_1
$\mathfrak{M}_{156}(0,1,2:3)$	$\{4\}$	D_2	S_1
$\mathfrak{M}_{156}(1,1,2:1)$	$\{17\}$	D_2	S_1
$\mathfrak{M}_{156}(1,1,2:2)$	$\{14\}$	D_2	S_1
$\mathfrak{M}_{166}(1,1,2:1)$	$\{8\}$	S_4	S_1
$\mathfrak{M}_{166}(1,1,2:2)$	$\{14\}$	S_4	S_1
$\mathfrak{M}_{166}(1,1,2:3)$	$\{17\}$	S_4	S_1
$\mathfrak{M}_{166}(2,1,1:1)$	$\{23\}$	S_3	S_1
$\mathfrak{M}_{166}(2,1,1:2)$	$\{10\}$	S_3	S_1

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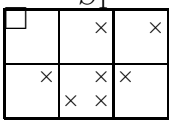
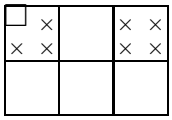
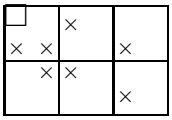
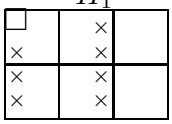
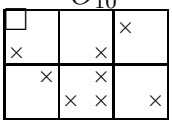
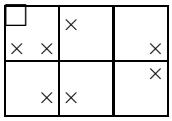
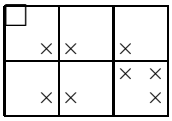
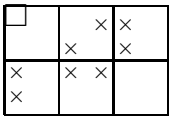
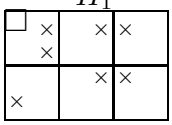
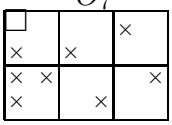
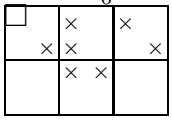
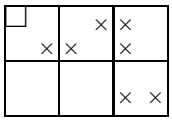
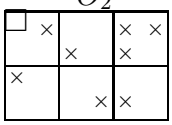
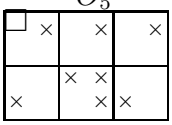
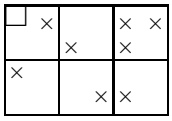
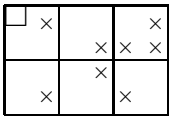
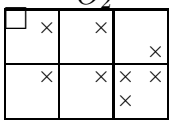
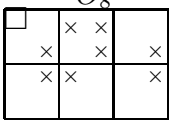
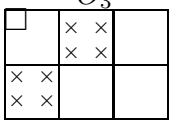
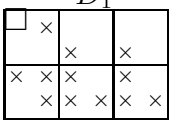
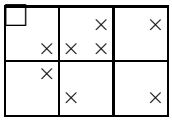
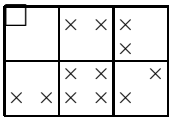
$\mathfrak{M}_{222}(0,0,0:1)$	$\{17,11\}$	$\{23,8\}$	$\{3,20\}$
$\mathfrak{M}_{222}(0,0,0:2)$	$\{15,17\}$	$\{23,8\}$	$\{3,20\}$
$\mathfrak{M}_{223}(0,1,0:1)$	$\{4,18\}$	$\{17,19\}$	H_1
$\mathfrak{M}_{223}(0,1,0:2)$	$\{12,18\}$	$\{11,17\}$	H_1
$\mathfrak{M}_{224}(0,0,2:1)$	$\{8,23\}$	$\{4,13\}$	O_2
$\mathfrak{M}_{224}(0,2,0:2)$	$\{4,10\}$	$\{5,18\}$	O_2
$\mathfrak{M}_{224}(0,0,0:1)$	$\{5,22\}$	$\{19,20\}$	O_2
$\mathfrak{M}_{224}(0,0,0:2)$	$\{1,12\}$	$\{14,19\}$	O_2
$\mathfrak{M}_{224}(0,1,1:1)$	$\{9,13\}$	$\{7,20\}$	O_2

$\mathfrak{M}_{224}(0,1,1:2)$	$\{9,13\}$	$\{16,23\}$	O_2
$\mathfrak{M}_{225}(0,2,2:1)$	$\{8,23\}$	$\{11,17\}$	D_1
$\mathfrak{M}_{225}(0,2,2:2)$	$\{9,22\}$	$\{11,17\}$	D_1
$\mathfrak{M}_{225}(0,2,0:1)$	$\{8,23\}$	$\{4,13\}$	D_1
$\mathfrak{M}_{225}(0,2,0:2)$	$\{17,23\}$	$\{4,13\}$	D_1
$\mathfrak{M}_{226}(0,1,2:1)$	$\{8,14\}$	$\{17,20\}$	S_1
$\mathfrak{M}_{226}(0,1,2:2)$	$\{8,14\}$	$\{11,17\}$	S_1
$\mathfrak{M}_{226}(0,2,4:1)$	$\{2,15\}$	$\{8,19\}$	S_1
$\mathfrak{M}_{226}(0,2,4:2)$	$\{2,21\}$	$\{3,12\}$	S_1
$\mathfrak{M}_{226}(0,2,2:1)$	$\{2,5\}$	$\{4,13\}$	S_1
$\mathfrak{M}_{226}(0,2,2:2)$	$\{6,18\}$	$\{8,17\}$	S_1
$\mathfrak{M}_{233}(0,0,3:1)$	$\{3,15\}$	H_{11}	
$\mathfrak{M}_{233}(0,0,3:2)$	$\{3,15\}$		
$\mathfrak{M}_{234}(0,0,4:1)$	$\{11,13\}$	H_1	O_{10}
$\mathfrak{M}_{234}(0,0,4:2)$	$\{14,17\}$		
$\mathfrak{M}_{234}(1,0,4:1)$	$\{14,17\}$		
$\mathfrak{M}_{234}(1,0,4:2)$	$\{14,17\}$	H_{12}	
$\mathfrak{M}_{234}(0,0,2:1)$	$\{11,17\}$	H_1	O_7
$\mathfrak{M}_{234}(0,0,2:2)$	$\{6,21\}$		
$\mathfrak{M}_{234}(1,0,2:1)$	$\{14,16\}$	H_1	O_7
$\mathfrak{M}_{234}(1,0,2:2)$	$\{14,17\}$	H_1	O_7
$\mathfrak{M}_{235}(0,2,2:1)$	$\{11,17\}$	H_5	D_1
$\mathfrak{M}_{235}(0,2,2:2)$	$\{5,17\}$	H_5	D_1
$\mathfrak{M}_{235}(1,2,4:1)$	$\{8,23\}$	H_6	D_1
$\mathfrak{M}_{235}(1,2,4:2)$	$\{3,8\}$	H_6	D_1
$\mathfrak{M}_{235}(0,1,4:1)$	$\{7,17\}$	H_6	D_1
$\mathfrak{M}_{235}(0,1,4:2)$	$\{16,17\}$	H_6	D_1
$\mathfrak{M}_{236}(0,2,3:1)$	$\{10,20\}$		S_1

$\mathfrak{M}_{236}(0,2,3:2)$	$\{2,9\}$		S_1
$\mathfrak{M}_{236}(0,2,3:3)$	$\{9,10\}$	H_{13}	S_1
$\mathfrak{M}_{236}(0,4,2:1)$	$\{3,5\}$	H_7	S_1
$\mathfrak{M}_{236}(0,4,2:2)$	$\{3,10\}$	H_7	S_1
$\mathfrak{M}_{236}(0,2,4:1)$	$\{5,10\}$	H_{14}	S_1
$\mathfrak{M}_{244}(2,0,2:1)$	$\{10,17\}$	O_2	O_5
$\mathfrak{M}_{244}(2,0,2:2)$	$\{11,17\}$	O_2	O_5
$\mathfrak{M}_{244}(0,0,4:1)$	$\{8,23\}$		
$\mathfrak{M}_{244}(0,0,4:2)$	$\{8,23\}$		
$\mathfrak{M}_{244}(0,0,4:3)$	$\{8,23\}$	O_2	O_8
$\mathfrak{M}_{244}(1,0,4:1)$	$\{17,23\}$	O_2	O_8
$\mathfrak{M}_{244}(1,0,4:2)$	$\{8,23\}$		
$\mathfrak{M}_{244}(1,0,2:1)$	$\{14,17\}$	O_2	O_5
$\mathfrak{M}_{244}(1,0,2:2)$	$\{15,17\}$	O_2	O_5
$\mathfrak{M}_{245}(2,2,6:1)$	$\{3,6\}$	O_9	D_1
$\mathfrak{M}_{245}(2,2,6:2)$	$\{3,15\}$	O_9	D_1
$\mathfrak{M}_{245}(0,2,6:1)$	$\{17,11\}$	O_9	D_1
$\mathfrak{M}_{245}(0,2,6:2)$	$\{8,17\}$	O_9	D_1
$\mathfrak{M}_{245}(0,2,2:1)$	$\{8,23\}$	O_2	D_1
$\mathfrak{M}_{245}(0,2,2:2)$	$\{20,23\}$	O_2	D_1
$\mathfrak{M}_{245}(2,1,4:1)$	$\{19,22\}$	O_3	D_1
$\mathfrak{M}_{245}(2,1,4:2)$	$\{1,22\}$	O_3	D_1
$\mathfrak{M}_{245}(0,1,2:1)$	$\{21,23\}$	O_2	D_1
$\mathfrak{M}_{245}(0,1,2:2)$	$\{14,23\}$	O_2	D_1
$\mathfrak{M}_{245}(0,0,4:1)$	$\{4,14\}$	O_3	D_1
$\mathfrak{M}_{245}(0,0,4:2)$	$\{4,14\}$	O_3	D_1
$\mathfrak{M}_{245}(0,0,4:2)$	$\{8,23\}$		
$\mathfrak{M}_{245}(1,0,4:1)$	$\{8,23\}$		
$\mathfrak{M}_{245}(1,0,4:2)$	$\{4,19\}$	O_3	D_1
$\mathfrak{M}_{245}(0,1,6:1)$	$\{4,17\}$	O_9	D_1
$\mathfrak{M}_{245}(0,1,6:2)$	$\{14,17\}$	O_9	D_1

$\mathfrak{M}_{245}(1,2,4:1)$	$\{11,22\}$	O_3	D_1
$\mathfrak{M}_{245}(1,2,4:2)$	$\{22,23\}$	O_3	D_1
$\mathfrak{M}_{245}(0,1,4:1)$	$\{4,8\}$	O_3	D_1
$\mathfrak{M}_{245}(0,1,4:2)$	$\{8,16\}$	O_3	D_1
$\mathfrak{M}_{246}(0,2,2:1)$	$\{14,16\}$	O_4	S_1
$\mathfrak{M}_{246}(0,2,2:2)$	$\{2,10\}$	O_4	S_1
$\mathfrak{M}_{246}(0,2,2:3)$	$\{10,18\}$	O_4	S_1
$\mathfrak{M}_{246}(2,2,3:1)$	$\{8,19\}$	O_5	S_1
$\mathfrak{M}_{246}(2,2,3:2)$	$\{1,19\}$	O_5	S_1
$\mathfrak{M}_{246}(1,2,3:1)$	$\{14,19\}$	O_5	S_1
$\mathfrak{M}_{246}(1,2,3:2)$	$\{4,14\}$	O_5	S_1
$\mathfrak{M}_{246}(1,2,5:1)$	$\{3,17\}$	O_2	S_2
$\mathfrak{M}_{246}(1,2,5:2)$	$\{14,17\}$	O_2	S_2
$\mathfrak{M}_{246}(1,2,2:1)$	$\{8,17\}$	O_4	S_1
$\mathfrak{M}_{246}(1,2,2:2)$	$\{7,17\}$	O_4	S_1
$\mathfrak{M}_{246}(0,2,3:1)$	$\{14,17\}$	O_5	S_1
$\mathfrak{M}_{246}(0,2,3:2)$	$\{7,14\}$	O_5	S_1
$\mathfrak{M}_{255}(0,0,8:1)$	$\{4,13\}$	D_1	D_6
$\mathfrak{M}_{255}(0,0,8:2)$	$\{4,14\}$	D_1	D_6
$\mathfrak{M}_{255}(0,0,8:3)$	$\{4,16\}$	D_1	D_6
$\mathfrak{M}_{255}(2,1,8:1)$	$\{17,22\}$	D_1	D_6
$\mathfrak{M}_{255}(2,1,8:2)$	$\{22,23\}$	D_1	D_6
$\mathfrak{M}_{255}(0,1,6:1)$	$\{1,14\}$	D_1	D_7
$\mathfrak{M}_{255}(0,1,6:2)$	$\{4,14\}$	D_1	D_7
$\mathfrak{M}_{255}(1,2,6:1)$	$\{14,22\}$	D_1	D_7
$\mathfrak{M}_{255}(1,2,6:2)$	$\{14,17\}$	D_1	D_7
$\mathfrak{M}_{256}(1,2,5:1)$	$\{7,17\}$	D_5	S_1
$\mathfrak{M}_{256}(1,2,5:2)$	$\{14,17\}$	D_5	S_1
$\mathfrak{M}_{256}(0,2,1:1)$	$\{4,12\}$	D_1	S_1
$\mathfrak{M}_{256}(0,2,1:2)$	$\{4,14\}$	D_1	S_1
$\mathfrak{M}_{256}(1,1,2:1)$	$\{10,17\}$	D_2	S_1
$\mathfrak{M}_{256}(1,1,2:2)$	$\{14,18\}$	D_2	S_1
$\mathfrak{M}_{256}(2,1,2:1)$	$\{4,16\}$	D_2	S_1
$\mathfrak{M}_{256}(2,1,2:2)$	$\{7,11\}$	D_2	S_1
$\mathfrak{M}_{256}(2,4,1:1)$	$\{17,23\}$	D_1	S_1
$\mathfrak{M}_{256}(2,4,1:2)$	$\{9,23\}$	D_1	S_1
$\mathfrak{M}_{256}(1,2,1:1)$	$\{13,17\}$	D_1	S_1
$\mathfrak{M}_{256}(1,2,1:2)$	$\{4,8\}$	D_1	S_1
$\mathfrak{M}_{256}(1,2,2:1)$	$\{11,14\}$	D_2	S_1
$\mathfrak{M}_{256}(1,2,2:2)$	$\{4,14\}$	D_2	S_1
$\mathfrak{M}_{256}(0,2,2:1)$	$\{13,14\}$	D_2	S_1
$\mathfrak{M}_{256}(0,2,2:2)$	$\{14,17\}$	D_2	S_1
$\mathfrak{M}_{256}(2,2,3:1)$	$\{4,8\}$	D_3	S_1
$\mathfrak{M}_{256}(2,2,3:2)$	$\{8,16\}$	D_3	S_1

$\mathfrak{M}_{256}(0,2,4:1)$	$\{14,17\}$	D_4	S_1
$\mathfrak{M}_{256}(0,2,4:2)$	$\{14,19\}$	D_4	S_1
$\mathfrak{M}_{256}(1,2,4:1)$	$\{11,19\}$	D_4	S_1
$\mathfrak{M}_{256}(1,2,4:2)$	$\{11,14\}$	D_4	S_1
$\mathfrak{M}_{256}(0,4,2:1)$	$\{17,23\}$	D_2	S_1
$\mathfrak{M}_{256}(0,4,2:2)$	$\{14,23\}$	D_2	S_1
$\mathfrak{M}_{266}(2,2,2:1)$	$\{14,17\}$	S_1	S_4
$\mathfrak{M}_{266}(2,2,2:2)$	$\{14,13\}$	S_1	S_4

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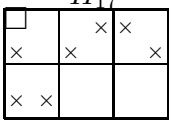
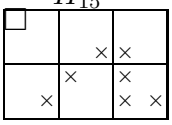
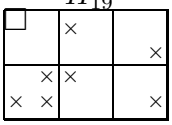
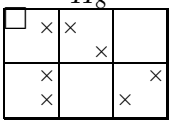
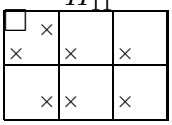
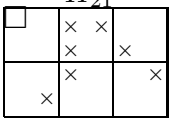
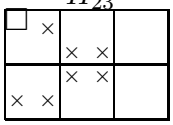
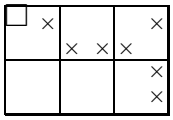
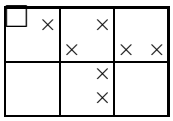
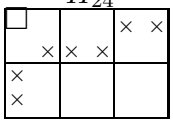
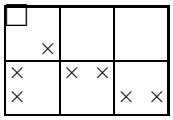
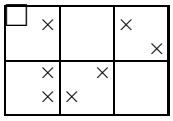
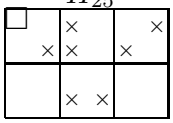
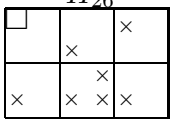
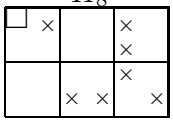
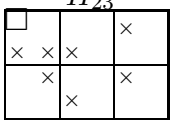
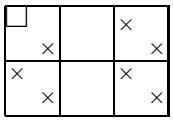
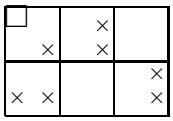
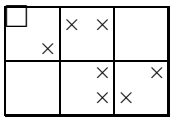
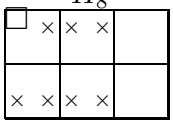
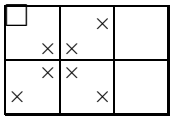
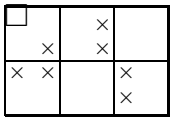
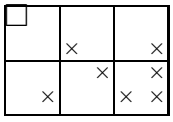
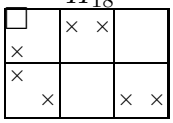
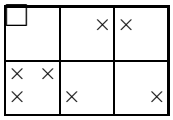
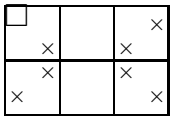
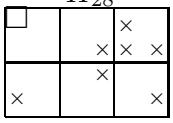
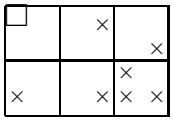
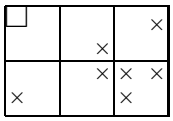
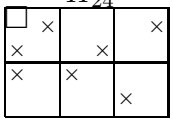
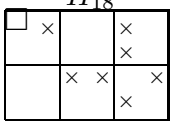
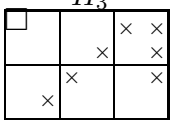
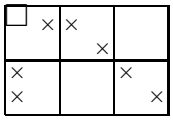
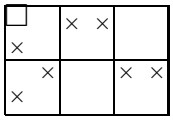
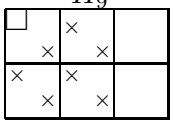
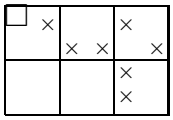
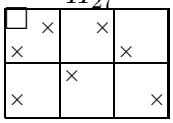
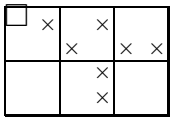
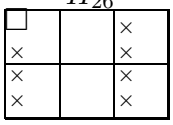
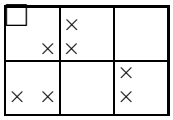
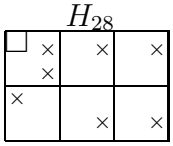
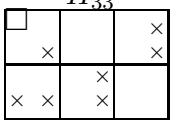
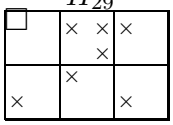
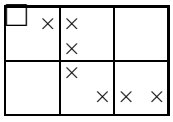
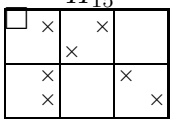
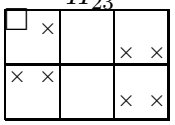
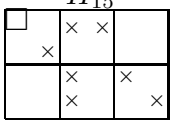
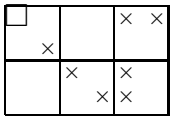
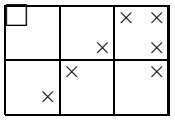
$\mathfrak{M}_{333}(3,3,1:1)$	H_1	H_{10}	H_8
$\mathfrak{M}_{333}(3,1,3:2)$	H_1	H_{10}	H_{17}
$\mathfrak{M}_{333}(3,3,3:1)$	H_1	H_{10}	
$\mathfrak{M}_{333}(3,3,3:2)$	H_1	H_{10}	H_9
$\mathfrak{M}_{334}(3,4,4:1)$	H_{10}	H_1	O_{10}
$\mathfrak{M}_{334}(3,4,4:2)$	H_9	H_1	O_{10}
$\mathfrak{M}_{334}(1,2,2:1)$	H_{15}	H_1	O_7
$\mathfrak{M}_{334}(1,2,2:2)$		H_1	O_7
$\mathfrak{M}_{334}(3,2,4:1)$	H_{16}	H_1	O_{10}
$\mathfrak{M}_{334}(3,2,4:2)$	H_{14}	H_1	O_{10}
$\mathfrak{M}_{334}(1,2,4:1)$	H_{18}	H_1	O_{10}
$\mathfrak{M}_{334}(1,2,4:2)$	H_{19}	H_1	O_{10}
$\mathfrak{M}_{334}(3,2,2:1)$		H_1	O_7
$\mathfrak{M}_{334}(3,2,2:2)$	H_8	H_1	O_7
$\mathfrak{M}_{335}(3,4,6:1)$		H_1	D_1
$\mathfrak{M}_{335}(3,4,6:2)$	H_8	H_1	D_1
$\mathfrak{M}_{335}(3,2,4:1)$	H_{11}	H_6	D_1
$\mathfrak{M}_{335}(3,2,4:2)$		H_6	D_1
$\mathfrak{M}_{335}(3,4,4:1)$	H_{20}	H_6	D_1
$\mathfrak{M}_{335}(3,4,4:2)$	H_{21}	H_6	D_1
$\mathfrak{M}_{335}(3,4,4:3)$		H_6	D_1
$\mathfrak{M}_{335}(3,4,4:4)$	H_9	H_6	D_1

$\mathfrak{M}_{335}(1,2,4:1)$	H_{22}	H_6	D_1
$\mathfrak{M}_{335}(1,2,4:2)$	H_{12}	H_6	D_1
$\mathfrak{M}_{335}(3,2,2:1)$	H_{23}	H_5	D_1
$\mathfrak{M}_{335}(3,2,2:2)$		H_5	D_1
$\mathfrak{M}_{336}(3,3,2:1)$		H_2	S_1
$\mathfrak{M}_{336}(3,3,2:2)$		H_2	S_1
$\mathfrak{M}_{336}(3,3,2:3)$	H_{24}	H_2	S_1
$\mathfrak{M}_{336}(1,4,3:1)$		H_3	S_1
$\mathfrak{M}_{336}(1,4,3:2)$		H_3	S_1
$\mathfrak{M}_{336}(1,3,2:1)$		H_2	S_1
$\mathfrak{M}_{336}(1,3,2:2)$	H_{25}	H_2	S_1
$\mathfrak{M}_{344}(4,4,2:1)$		O_2	O_5
$\mathfrak{M}_{344}(4,4,2:2)$	H_{26}	O_2	O_5
$\mathfrak{M}_{344}(4,4,4:1)$		O_2	O_8
$\mathfrak{M}_{344}(4,4,4:2)$	H_{24}	O_2	O_8
$\mathfrak{M}_{344}(2,4,2:1)$	H_{15}	O_2	O_5
$\mathfrak{M}_{344}(4,2,2:2)$	H_8	O_2	O_5
$\mathfrak{M}_{344}(2,4,4:1)$		O_2	O_8
$\mathfrak{M}_{344}(2,4,4:2)$	H_{23}	O_2	O_8
$\mathfrak{M}_{344}(2,2,4:1)$		O_2	O_8
$\mathfrak{M}_{344}(2,2,4:2)$	H_{25}	O_2	O_8
$\mathfrak{M}_{344}(2,2,4:3)$	H_{27}	O_2	O_8

$\mathfrak{M}_{345}(4,6,6:1)$		O_9	D_1
$\mathfrak{M}_{345}(4,6,6:2)$		O_9	D_1
$\mathfrak{M}_{345}(4,4,2:1)$		O_2	D_1
$\mathfrak{M}_{345}(4,4,2:2)$	H_8	O_2	D_1
$\mathfrak{M}_{345}(0,4,4:1)$		O_3	D_1
$\mathfrak{M}_{345}(0,4,4:2)$		O_3	D_1
$\mathfrak{M}_{345}(4,4,6:1)$		O_9	D_1
$\mathfrak{M}_{345}(4,4,6:2)$		O_9	D_1
$\mathfrak{M}_{345}(4,2,2:1)$	H_{26}	O_2	D_1
$\mathfrak{M}_{345}(4,2,2:2)$	H_{18}	O_2	D_1
$\mathfrak{M}_{345}(2,6,4:1)$		O_3	D_1
$\mathfrak{M}_{345}(2,6,4:2)$		O_3	D_1
$\mathfrak{M}_{345}(4,4,4:1)$		O_3	D_1
$\mathfrak{M}_{345}(4,4,4:2)$	H_{28}	O_3	D_1
$\mathfrak{M}_{345}(4,4,4:3)$		O_3	D_1
$\mathfrak{M}_{345}(4,4,4:4)$		O_3	D_1
$\mathfrak{M}_{345}(4,2,4:1)$	H_{25}	O_3	D_1

$\mathfrak{M}_{345}(4,2,4:2)$		O_3	D_1
$\mathfrak{M}_{345}(2,2,4:1)$	H_{24}	O_3	D_1
$\mathfrak{M}_{345}(2,2,4:2)$		O_3	D_1
$\mathfrak{M}_{345}(2,4,4:1)$	H_{23}	O_3	D_1
$\mathfrak{M}_{345}(2,4,4:2)$	H_{29}	O_3	D_1
$\mathfrak{M}_{346}(4,2,2:1)$	H_2	O_4	S_1
$\mathfrak{M}_{346}(4,2,2:2)$	H_{18}	O_4	S_1
$\mathfrak{M}_{346}(2,3,2:1)$		O_4	S_1
$\mathfrak{M}_{346}(2,3,2:2)$	H_{17}	O_4	S_1
$\mathfrak{M}_{346}(4,3,2:1)$	H_3	O_4	S_1
$\mathfrak{M}_{346}(4,3,2:2)$		O_4	S_1
$\mathfrak{M}_{346}(2,4,2:1)$		O_4	S_1
$\mathfrak{M}_{346}(2,4,2:2)$		O_4	S_1
$\mathfrak{M}_{346}(2,2,2:1)$	H_9	O_4	S_1
$\mathfrak{M}_{346}(2,2,2:2)$		O_4	S_1
$\mathfrak{M}_{346}(2,3,5:1)$		O_2	S_2
$\mathfrak{M}_{346}(2,3,5:2)$	H_{23}	O_2	S_2
$\mathfrak{M}_{346}(2,4,3:1)$	H_{27}	O_5	S_1
$\mathfrak{M}_{346}(2,4,3:2)$		O_5	S_1
$\mathfrak{M}_{346}(2,3,3:1)$		O_5	S_1
$\mathfrak{M}_{346}(2,3,3:2)$	H_{24}	O_5	S_1
$\mathfrak{M}_{355}(2,6,4:1)$	H_{15}	D_1	D_8

$\mathfrak{M}_{355}(2,6,4:2)$	H_{26}	D_1	D_8
$\mathfrak{M}_{355}(4,6,8:1)$		D_1	D_6
$\mathfrak{M}_{355}(4,6,8:2)$		D_1	D_6
$\mathfrak{M}_{355}(2,2,8:1)$	H_{30}	D_1	D_6
$\mathfrak{M}_{355}(2,2,8:2)$	H_{31}	D_1	D_6
$\mathfrak{M}_{355}(2,2,8:3)$	H_{26}	D_1	D_6
$\mathfrak{M}_{355}(4,4,8:1)$	H_{13}	D_1	D_6
$\mathfrak{M}_{355}(4,4,8:2)$	H_{32}	D_1	D_6
$\mathfrak{M}_{355}(4,4,4:1)$	H_{13}	D_1	D_8
$\mathfrak{M}_{355}(4,4,4:2)$	H_{32}	D_1	D_8
$\mathfrak{M}_{355}(2,4,6:1)$	H_{23}	D_1	D_7
$\mathfrak{M}_{355}(2,4,6:2)$	H_{28}	D_1	D_7
$\mathfrak{M}_{355}(4,4,6:1)$		D_1	D_7
$\mathfrak{M}_{355}(4,4,6:2)$	H_{33}	D_1	D_7
$\mathfrak{M}_{356}(2,2,5:1)$		D_5	S_1
$\mathfrak{M}_{356}(2,2,5:2)$	H_{20}	D_5	S_1
$\mathfrak{M}_{356}(2,3,1:1)$	H_{11}	D_1	S_1
$\mathfrak{M}_{356}(2,3,1:2)$	H_{15}	D_1	S_1
$\mathfrak{M}_{356}(2,3,1:3)$	H_{22}	D_1	S_1
$\mathfrak{M}_{356}(4,3,1:1)$	H_{21}	D_1	S_1
$\mathfrak{M}_{356}(4,3,1:2)$	H_{19}	D_1	S_1
$\mathfrak{M}_{356}(2,3,2:1)$	H_{31}	D_3	S_1
$\mathfrak{M}_{356}(2,3,2:2)$	H_{30}	D_3	S_1
$\mathfrak{M}_{356}(2,2,2:1)$	H_{29}	D_2	S_1
$\mathfrak{M}_{356}(2,2,2:2)$		D_2	S_1
$\mathfrak{M}_{356}(4,3,2:1)$		D_2	S_1
$\mathfrak{M}_{356}(4,3,2:2)$	H_{24}	D_2	S_1
$\mathfrak{M}_{356}(4,3,3:1)$	H_{15}	D_3	S_1
$\mathfrak{M}_{356}(4,3,3:2)$		D_3	S_1

$\mathfrak{M}_{356}(4,2,5:1)$	H_{16}	D_5	S_1
$\mathfrak{M}_{356}(4,2,5:2)$	H_9	D_5	S_1
$\mathfrak{M}_{356}(4,2,4:1)$	H_{23}	D_4	S_1
$\mathfrak{M}_{356}(4,2,4:2)$		D_4	S_1
$\mathfrak{M}_{356}(4,2,1:1)$	H_{20}	D_1	S_1
$\mathfrak{M}_{356}(4,2,1:2)$	H_9	D_1	S_1
$\mathfrak{M}_{366}(2,2,2:1)$	H_{33}	S_1	S_4
$\mathfrak{M}_{366}(2,2,2:2)$	H_{23}	S_1	S_4
$\mathfrak{M}_{366}(3,4,1:1)$	H_{15}	S_1	S_3
$\mathfrak{M}_{366}(3,4,1:2)$		S_1	S_3
$\mathfrak{M}_{366}(3,3,1:1)$		S_1	S_3
$\mathfrak{M}_{366}(3,3,1:2)$		S_1	S_3

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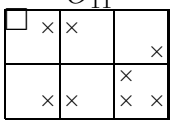
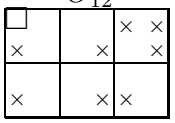
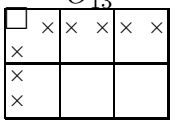
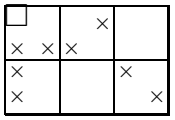
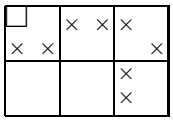
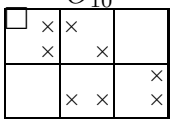
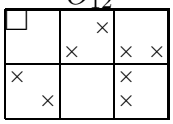
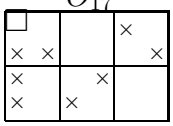
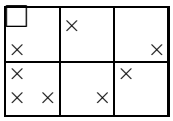
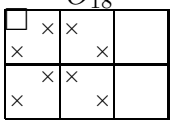
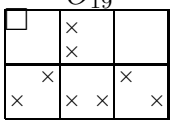
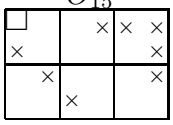
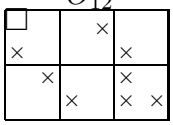
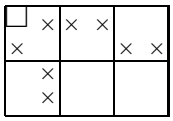
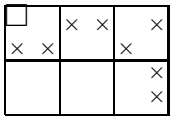
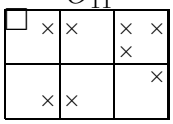
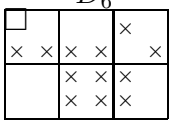
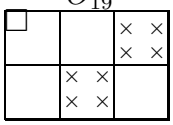
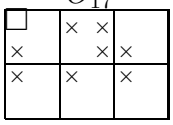
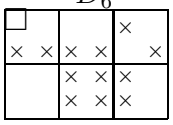
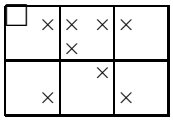
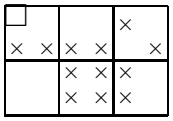
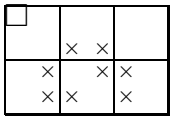
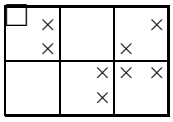
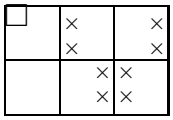
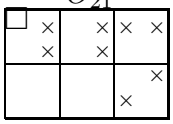
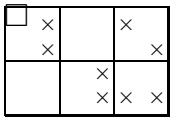
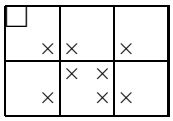
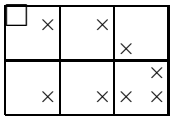
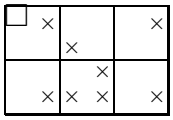
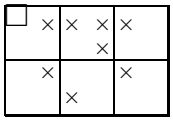
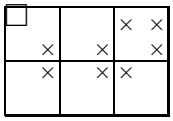
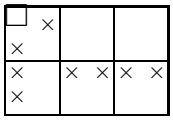
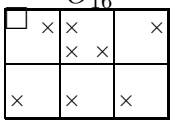
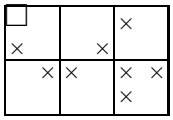
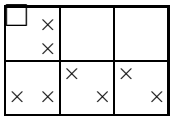
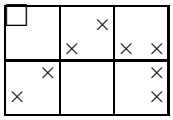
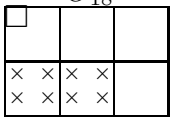
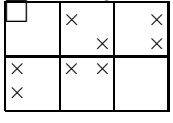
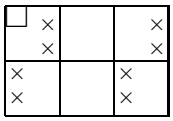
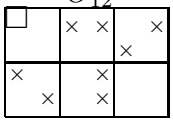
$\mathfrak{M}_{444}(2,2,4:1)$	O_{11}	O_2	O_8
$\mathfrak{M}_{444}(2,2,4:2)$		O_2	O_8
$\mathfrak{M}_{444}(2,2,4:3)$	O_{12}	O_2	O_8
$\mathfrak{M}_{444}(4,2,4:1)$		O_2	O_8
$\mathfrak{M}_{444}(4,2,4:2)$	O_{13}	O_2	O_8
$\mathfrak{M}_{445}(2,6,4:1)$		O_3	D_1
$\mathfrak{M}_{445}(2,6,4:2)$		O_3	D_1
$\mathfrak{M}_{445}(4,2,2:1)$		O_2	D_1
$\mathfrak{M}_{445}(4,2,2:2)$	O_{14}	O_2	D_1

$\mathfrak{M}_{445}(4,6,4:1)$		O_3	D_1
$\mathfrak{M}_{445}(4,6,4:2)$	O_{15}	O_3	D_1
$\mathfrak{M}_{445}(4,6,6:1)$	O_{15}	O_9	D_1
$\mathfrak{M}_{445}(4,6,6:2)$	O_{11}	O_9	D_1
$\mathfrak{M}_{445}(4,4,2:1)$	O_{16}	O_2	D_1
$\mathfrak{M}_{445}(4,4,2:2)$	O_{10}	O_2	D_1
$\mathfrak{M}_{445}(4,4,2:3)$		O_2	D_1
$\mathfrak{M}_{445}(4,4,4:1)$	O_{12}	O_3	D_1
$\mathfrak{M}_{445}(4,4,4:2)$		O_3	D_1
$\mathfrak{M}_{445}(2,4,4:1)$	O_{16}	O_3	D_1
$\mathfrak{M}_{445}(2,4,4:2)$	O_{17}	O_3	D_1
$\mathfrak{M}_{446}(2,4,3:1)$		O_5	S_1
$\mathfrak{M}_{446}(2,4,3:2)$		O_5	S_1
$\mathfrak{M}_{446}(2,5,2:1)$	O_{18}	O_4	S_1
$\mathfrak{M}_{446}(2,5,2:2)$		O_4	S_1
$\mathfrak{M}_{446}(4,3,5:1)$	O_{19}	O_2	S_2
$\mathfrak{M}_{446}(4,3,5:2)$		O_2	S_2
$\mathfrak{M}_{446}(4,3,5:3)$	O_{20}	O_2	S_2
$\mathfrak{M}_{446}(4,3,2:1)$	O_{15}	O_4	S_1
$\mathfrak{M}_{446}(4,3,2:2)$		O_4	S_1
$\mathfrak{M}_{446}(2,3,2:1)$	O_{12}	O_4	S_1
$\mathfrak{M}_{446}(2,3,2:2)$		O_4	S_1
$\mathfrak{M}_{455}(2,6,4:1)$	O_5	D_1	D_8
$\mathfrak{M}_{455}(2,6,4:2)$	O_{20}	D_1	D_8

$\mathfrak{M}_{455}(6,6,8:1)$		D_1	D_6
$\mathfrak{M}_{455}(6,6,8:2)$		D_1	D_6
$\mathfrak{M}_{455}(6,6,8:3)$	O_{11}	D_1	D_6
$\mathfrak{M}_{455}(4,2,4:1)$		D_1	
$\mathfrak{M}_{455}(2,4,4:2)$	O_{19}	D_1	D_8
$\mathfrak{M}_{455}(2,2,8:1)$		D_1	D_6
$\mathfrak{M}_{455}(2,2,8:2)$	O_{14}	D_1	D_6
$\mathfrak{M}_{455}(4,4,8:1)$	O_{16}	D_1	D_6
$\mathfrak{M}_{455}(4,4,8:2)$	O_{17}	D_1	D_6
$\mathfrak{M}_{455}(4,4,4:1)$		D_1	
$\mathfrak{M}_{455}(4,4,4:2)$		D_1	
$\mathfrak{M}_{456}(2,2,1:1)$		D_1	S_1
$\mathfrak{M}_{456}(2,2,1:2)$		D_1	S_1
$\mathfrak{M}_{456}(2,2,1:3)$		D_1	S_1
$\mathfrak{M}_{456}(2,3,1:1)$	O_{22}	D_1	S_1
$\mathfrak{M}_{456}(2,3,1:2)$	O_{13}	D_1	S_1
$\mathfrak{M}_{456}(6,5,1:1)$	O_9	D_1	S_1
$\mathfrak{M}_{456}(6,5,1:2)$	O_{21}	D_1	S_1
$\mathfrak{M}_{456}(4,2,1:1)$		D_1	S_1
$\mathfrak{M}_{456}(4,2,1:2)$		D_1	S_1

$\mathfrak{M}_{456}(6,2,2:1)$		D_2	S_1
$\mathfrak{M}_{456}(6,2,2:2)$		D_2	S_1
$\mathfrak{M}_{456}(6,2,2:3)$		D_2	S_1
$\mathfrak{M}_{456}(2,2,2:1)$		D_2	S_1
$\mathfrak{M}_{456}(2,2,2:2)$		D_2	S_1
$\mathfrak{M}_{456}(4,4,3:1)$		D_3	S_1
$\mathfrak{M}_{456}(4,4,3:2)$	O_{11}	D_3	S_1
$\mathfrak{M}_{456}(4,3,1:1)$	O_{12}	D_1	S_1
$\mathfrak{M}_{456}(4,3,1:2)$	O_{16}	D_1	S_1
$\mathfrak{M}_{456}(2,3,2:1)$		D_2	S_1
$\mathfrak{M}_{456}(2,3,2:2)$		D_2	S_1
$\mathfrak{M}_{456}(6,3,3:1)$		D_3	S_1
$\mathfrak{M}_{456}(6,3,3:2)$		D_3	S_1
$\mathfrak{M}_{456}(4,5,5:1)$	O_{21}	D_5	S_1
$\mathfrak{M}_{456}(4,5,5:2)$	O_{18}	D_5	S_1
$\mathfrak{M}_{456}(4,5,1:1)$		D_1	S_1
$\mathfrak{M}_{456}(4,5,1:2)$	O_{18}	D_1	S_1
$\mathfrak{M}_{456}(2,5,5:1)$		D_5	S_1

$\mathfrak{M}_{456}(2,5,5:2)$		D_5	S_1
$\mathfrak{M}_{466}(3,5,2:1)$	O_{12}	S_1	S_4
$\mathfrak{M}_{466}(3,5,2:2)$		S_1	S_4
$\mathfrak{M}_{466}(3,3,2:1)$	O_{16}	S_1	S_4
$\mathfrak{M}_{466}(3,3,2:2)$	O_{14}	S_1	S_4
$\mathfrak{M}_{466}(3,2,2:1)$	O_{22}	S_1	S_4
$\mathfrak{M}_{466}(3,2,2:2)$	O_{15}	S_1	S_4

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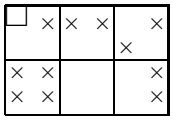
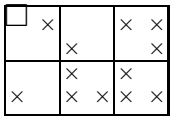
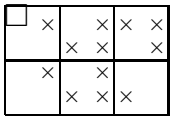
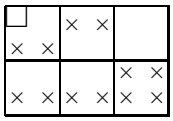
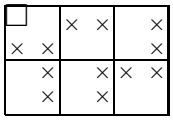
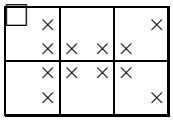
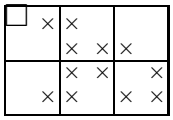
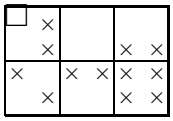
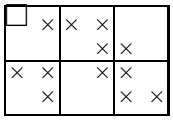
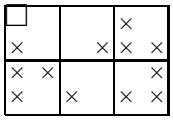
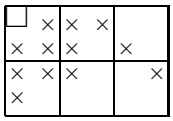
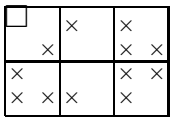
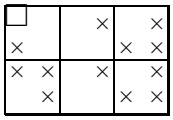
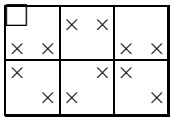
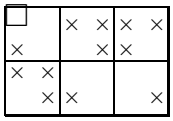
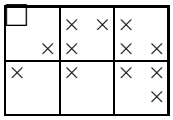
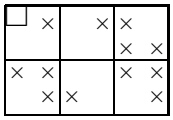
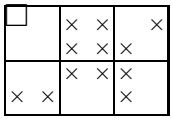
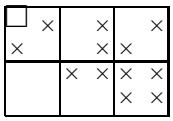
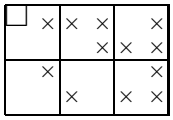
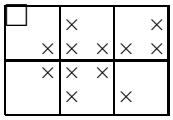
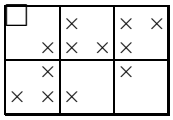
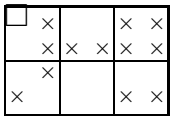
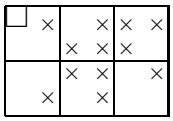
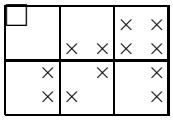
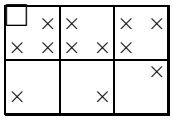
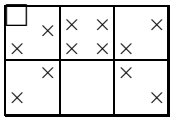
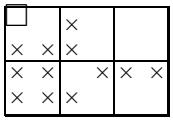
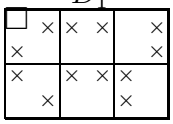
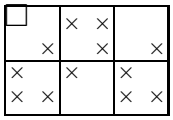
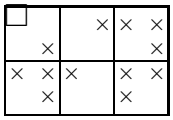
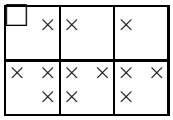
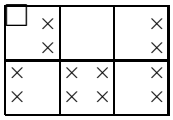
$\mathfrak{M}_{555}(4,4,8:1)$		D_1	D_6
$\mathfrak{M}_{555}(4,4,8:2)$		D_1	D_6
$\mathfrak{M}_{555}(4,4,8:3)$		D_1	D_6
$\mathfrak{M}_{555}(8,8,8:1)$		D_1	D_6
$\mathfrak{M}_{555}(8,8,8:2)$		D_1	D_6
$\mathfrak{M}_{556}(4,5,1:1)$		D_1	S_1
$\mathfrak{M}_{556}(4,5,1:2)$		D_1	S_1
$\mathfrak{M}_{556}(6,2,1:1)$		D_1	S_1
$\mathfrak{M}_{556}(6,2,1:2)$		D_1	S_1
$\mathfrak{M}_{556}(8,1,1:1)$		D_1	S_1

$\mathfrak{M}_{556}(8,1,1:2)$		D_1	S_1
$\mathfrak{M}_{556}(8,4,1:1)$		D_1	S_1
$\mathfrak{M}_{556}(8,4,1:2)$		D_1	S_1
$\mathfrak{M}_{556}(8,3,1:1)$		D_1	S_1
$\mathfrak{M}_{556}(8,3,1:2)$		D_1	S_1
$\mathfrak{M}_{556}(8,2,1:1)$		D_1	S_1
$\mathfrak{M}_{556}(8,2,1:2)$		D_1	S_1
$\mathfrak{M}_{556}(4,2,1:1)$		D_1	S_1
$\mathfrak{M}_{556}(4,2,1:2)$		D_1	S_1
$\mathfrak{M}_{556}(6,5,2:1)$		D_2	S_1
$\mathfrak{M}_{556}(6,5,2:2)$		D_2	S_1
$\mathfrak{M}_{556}(4,2,2:1)$		D_2	S_1
$\mathfrak{M}_{556}(4,2,2:2)$		D_2	S_1

$\mathfrak{M}_{556}(8,5,2:1)$		D_2	S_1
$\mathfrak{M}_{556}(8,5,2:2)$		D_2	S_1
$\mathfrak{M}_{556}(4,3,3:1)$		D_3	S_1
$\mathfrak{M}_{556}(4,3,3:2)$		D_3	S_1
$\mathfrak{M}_{566}(1,1,4:1)$		S_1	S_6
$\mathfrak{M}_{566}(1,1,4:2)$	D_1	S_1	S_6
$\mathfrak{M}_{566}(3,2,1:1)$		S_1	S_3
$\mathfrak{M}_{566}(3,2,1:2)$		S_1	S_3
$\mathfrak{M}_{566}(2,2,2:1)$		S_1	S_4
$\mathfrak{M}_{566}(2,2,2:2)$		S_1	S_4
$\mathfrak{M}_{566}(4,4,2:1)$		S_1	S_4
$\mathfrak{M}_{566}(4,4,2:2)$	D_9	S_1	S_4

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$\mathfrak{M}_{666}(2,2,1:1)$	S_9	S_1	S_3
$\mathfrak{M}_{666}(1,2,2:2)$	S_8	S_1	S_4

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