GROUND-STATE NUCLEAR PROPERTIES OF SOME RARE EARTH NUCLEI IN RELATIVISTIC MEAN FIELD THEORY

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In this study, rare earth nuclei, ^{160}Gd, ^{168}Er, ^{170}Er and isotopic chain of Dy were investigated using relativistic mean field theory with non-linear NL3 and NLSH parameters sets. Binding energies per nucleon, neutron radii, proton radii, charge radii, neutron and proton quadrupole moments of these nuclei were calculated. Also, these ground state properties were calculated using non-relativistic Hartree-Fock-Bogoliubov method with parameters set SKP. Predictions of this work were compared with available experimental data and some predictions calculated with different parameters set in relativistic mean field theory.

Relativistic models of the nucleus have attracted much attention in recent 30 years. One of them is relativistic mean field theory (RMF) [1, 2]. Several attempts have been made to describe the nuclear properties using RMF theory due to its advantages over the non-relativistic density-dependent Skyrme approaches [3]. The RMF theory has been successful in describing the ground-state properties of nuclei about both the line of stability [1, 4] and far away from the line of stability [5]. On the other hand, studies of rare earth nuclei which are heavy deformed are attracted [6- 8].

The aim of this study was investigated ground-state nuclear properties of some rare earth nuclei within the framework of RMF theory using NL3 and NLSH parameters sets and was compared these results with Hartree-Fock-Bogoliubov method results.

The ansatz of the interaction in the RMF theory is based upon Lagrangian density of the form [2]:

$$\begin{aligned}\mathcal{L} = &\bar{\psi}\left(i\partial\!\!\!/ - M\right)\psi + \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \tfrac{1}{2}\Omega_{\mu\nu}\Omega^{\mu\nu} \\ &+ \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \tfrac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \tfrac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu \\ &- \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\omega\!\!\!/\psi - g_\rho\bar{\psi}\vec{\rho\!\!\!/}\vec{\tau}\psi - e\bar{\psi}A\!\!\!/\psi\end{aligned} \tag{1}$$

where the Dirac nucleon interacts with the σ and the ω meson fields. The ρ meson generates the isovector component of the force. The Lagrangian contains a nonlinear scalar self-interaction of the σ meson.

$$U(\sigma) = \tfrac{1}{2}m_\sigma^2\sigma^2 + \tfrac{1}{3}g_2\sigma^3 + \tfrac{1}{4}g_3\sigma^4 \tag{2}$$

This term is necessary for appropriate description of surface properties. *M*, m_σ, m_ω and m_ρ are the nucleon, $\sigma-$, $\omega-$ and $\rho-$meson masses, respectively, while g_σ, g_ω, g_ρ and $e^2/4\pi = 1/137$ are the corresponding coupling constants for the mesons and the photon. The field tensors of the vector mesons and of the electromagnetic fields take the following form:

$$\Omega^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu \tag{3}$$

$$\vec{R}^{\mu\nu} = \partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu \tag{4}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{5}$$

The Dirac spinors ψ_i of the nucleon and the fields of $\sigma-$, $\omega-$, $\rho-$mesons are solutions of the coupled Dirac and Klein-Gordon equations via the classical variational principle and are then solved by the self-consistent method for axially symmetric systems of nucleons with additional pairing interaction. These equations are solved by iterative procedure; starting from an estimate of the meson and electromagnetic fields, one can solve the Dirac equation and obtain the spinors. These are used to obtain the densities and currents. They are used for solution of the Klein-Gordon equations and provide the new estimates of the meson and electromagnetic fields for the next iteration. This iteration is continued till the convergence up to the desired accuracy is achieved. When densities are calculated, negative-energy states are neglected (no-sea approximation), i.e. the vacuum is not polarized. Details are given in [7]. Some input parameters corresponding values of nucleon masses, mesons masses_and coupling constants are necessary such a calculation. In this study it was used NLSH [5] and NL3 [9] non-linear parameters sets. These sets are shown in Table1.

The rare earth nuclei considered here are even-mass nuclei and these nuclei are open-shell nuclei both in protons and neutrons, thus requiring the inclusion of pairing. The parameters sets NLSH and NL3 has been employed for these nuclei. The number of shells taken into account are 12 and 20 for the fermionic and bosonic expansions, respectively. The basis parameters $\hbar\omega$ and β_0 used for the calculations have been taken to be $41A^{\frac{1}{3}}$ and 0.3, respectively. In order to investigate these rare earth nuclei we have performed the calculations with Saxoon-Woods initial wavefunctions.

In this study, also these rare earth nuclei investigated in framework of Hartree-Fock-Bogoliubov method using SKP [10] parameters set for comparison. Ergo, HFBTHO computer code which is presented by Stoitsov et al. [11] was used. This code provides axially deformed solution of the Hartree-Fock-Bogoliubov [HFB] equations. For HFB calculations, β_0 basis parameters have been taken to be

0.3 as it is mentined above in relativistic mean field calculations.

Table1.The parameters sets NLSH [5] and NL3[9] used in the relativistic mean field Lagrangian.

Parameter	***NLSH***	***NL3***
M (MeV)	939.00	939.00
m_σ (MeV)	526.059	508.194
m_ω (MeV)	783.00	782.501
m_ρ (MeV)	763.00	763.000
g_σ	10.4444	10.217
g_ω	12.945	12.868
g_ρ	4.383	4.474
g_2	-6.9099	-10.431
g_3	-15.8337	-28.885

Our predictions of binding energy per nucleon and quadrupole deformation parameter β_2 of protons for isotopic chain of Dy in not only relativistic mean field theory with NLSH and NL3 parameters sets but also HFB method with SKP parameters set are shown in Fig.1 and Fig.2 respectively. Experimental curves are also shown for comparisons.

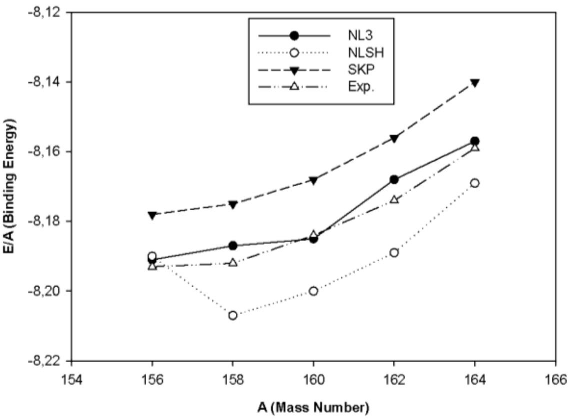

Fig.1 Binding energy per nucleon for Dy isotopes. Experimental values were obtained from [13].

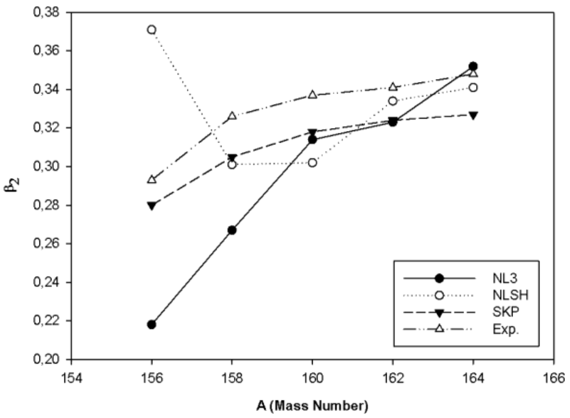

Fig.2 Quadrupole deformation parameter β_2 for Dy isotopes are shown as a function of mass number. Experimental values were obtained from [12].

As seen from the Fig.1, predictions of RMF theory with NL3 to the binding energy per nucleon for Dy isotopes are good agreement with experimental curve [13]. Maximal deviations between these values are approximately 0.01 MeV. Also predictions of the other sets are agreement with experimental value. The maximal deviations between the experimental values and the other sets are about 0.2 MeV. On the other hand, as seen from the Fig.2, predictions of HFB method using the parameters set SKP to quadrupole deformations are better agreement with experimental values than the other sets.

In Fig.3 and Fig.4, proton radii and charge radii for Dy isotopes are shown. In this figures, predictions of relativistic mean field theory with TMA parameters set [14] are also shown.

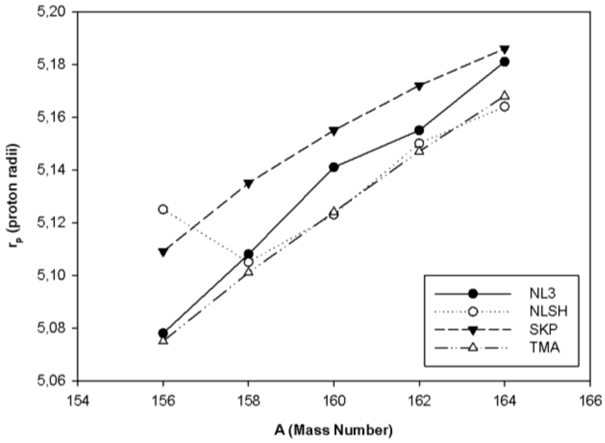

Fig.3 r_p, proton radii for Dy isotopes are shown as a function of mass number. Predictions of relativistic mean fielt theory with TMA parameters set were obtained from [14].

In Table 1, the binding energies per nucleon (E/A); proton, nötron and charge radii; quadrupole moments for neutron, proton and quadrupole deformation parameters β_2 are shown for ^{160}Gd, ^{168}Er and ^{170}Er nuclei. For comparison, predictions of NL2 parameters set [4] and available experimental values [12, 13] are shown. As seen from the Table 1, the binding energies per nucleon for ^{160}Gd, ^{168}Er and ^{170}Er nuclei are well described in both relativistic mean field theory with non-linear NL2 [4], NL3 and NLSH parameters sets and Skyrme Hartree-Fock-Bogoliubov method with SKP parameters set.

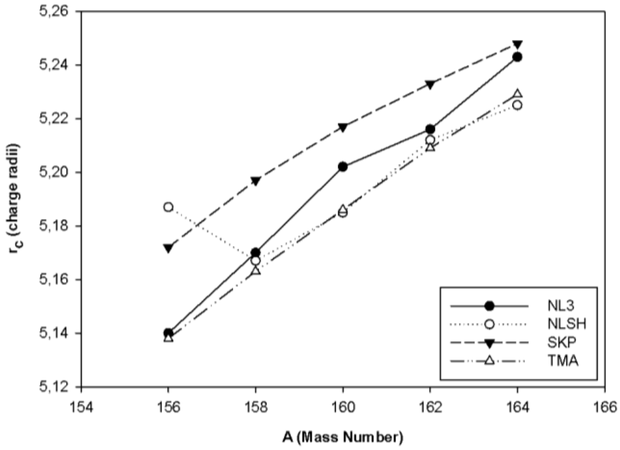

Fig.4 r_c, charge radii for Dy isotopes are shown as a function of mass number. Predictions of relativistic mean field theory with TMA parameters set are also shown [14].

The deviations between the experimental values and the predictions described above are approximately 0.2 MeV. However, for description of quadrupole deformation parameter β_2, predictions of relativistic mean field theory using NLSH set and predictions of Hartree-Fock-Bogoliubov method using SKP set are good agreement with experimental values

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Table 2. Calculations of ground-state properties to some rare earth nuclei in both relativistic mean field theory with NL3 and NLSH parameters sets and HFB method with SKP parameters set. Also, NL2 [4] and experimental values [12, 13] are shown .

Nucleus		E/A	r_n	r_p	r_c	Q_n	Q_p	β
^{160}Gd	NL3	-8.178	5.365	5.126	6.515	10.119	6.883	0.330
	NLSH	-8.197	5.336	5.120	6.515	10.386	7.111	0.340
	SKP	-8.167	5.279	5.143	5.205	10.795	7.408	0.330
	NL2	-8.18	5.37	5.11	5.17	9.59	6.41	0.31
	Exp.	-8.173					7.265	0.353
^{168}Er	NL3	-8.126	5.446	5.225	6.621	11.652	7.940	0.351
	NLSH	-8.136	5.410	5.210	6.621	11.309	7.756	0.342
	SKP	-8.112	5.351	5.228	5.289	11.333	7.826	0.322
	NL2	-8.15	5.44	5.20	5.26	10.44	7.05	0.31
	Exp.	-8.130					7.630	0.334
^{170}Er	NL3	-8.110	5.479	5.244	6.648	12.279	8.186	0.360
	NLSH	-8.121	5.449	5.228	6.648	11.962	7.956	0.356
	SKP	-8.095	5.377	5.242	5.302	11.645	7.871	0.321
	NL2	-8.11	5.47	5.21	5.27	10.90	7.19	0.32
	Exp.	-8.112					7.650	0.336

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