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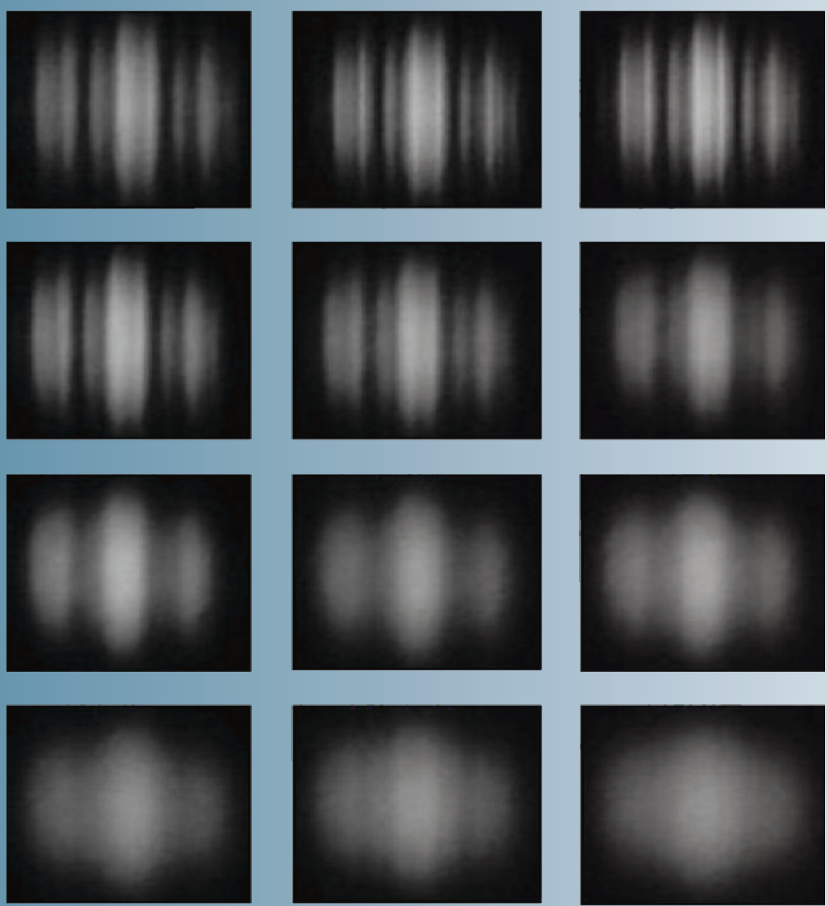

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CALCULATION OF FISSION BARRIER OF $^{230-234}$ Pa ISOTOPES

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The fission barriers of $^{230-234}$Pa have been carried out by using the BARRIER code developed by Garcia. The nuclear shape has been parameterized in terms of Cassini ovaloids proposed by Pashkevich. The single- particle energies have been calculated as function of the deformation parameters of an axially deformed Woods-Saxon potential, as input to the shell correction calculations. To obtain the total nuclear energy, it is also necessary to add a pairing energy in order to take into consideration the short range nuclear interactions.

I. INTRODUCTION

This work is a theoretical support for the fission barriers of ground states and excited states in $^{230-234}$Pa. The isotopes nuclei of $^{230-234}$Pa have distinct fission properties. Differences have been found in the equilibrium deformation, first and second minimum, and saddle points of these Pa isotopes. Asymmetric mass division was also found of ones.

In our work, the first steps to start the optimization of the potential parameters are to fix the appropriate value of the equilibrium deformation of the nucleus. In the shell model approach, this is achieved in most of the practical applications using the Strutinsky method [1]. In this work, the extremal points were calculated with the BARRIER code [2], which includes the Strutinsky method with the Pashkevich parametrization for the nuclear shape. In our calculations, the pairing energy was evaluated accordingly to the commonly used prescriptions of the BCS approach which is including blocking effect [3, 4].

II. METHOD OF CALCULATION

Fission Barrier

Nuclear Deformation

As a function of deformation, average values of various nuclear properties are only calculated by macroscopic-microscopic method in the Strutinskys formalism [1, 5]. Within this method, the total energy surface of nuclear system is given by [3]

$$V(N,Z,\beta) = E_{LD} + E_{Shell} + E_{Pair} \qquad (1)$$

The liquid drop model gives the macroscopic terms. The microscopic portion is given by E_{Shell} and E_{Pair}. In order to obtain the best results for the single-particle spectrum in the equilibrium deformation the total energy is minimized with respect to the deformation parameters. In this work we used the Cassini ovaloid shape parametrization. The equilibrium deformation parameters are the quadrupole moment term (ε) and the hexadecapole moment term (α_4).

Shell Correction Method

The fission barrier is concerned with surface energy and Coulomb energy terms. The sum of their contributions to the liquid drop energy relative to the energy of a spherical liquid drop can be written as

$$\begin{aligned} &E_{LD}(shape) - E_{LD}(0) = \\ &\{[f(shape)-1] + 2\chi[g(shape)-1]\}E_s(0) \end{aligned} \qquad (2)$$

where $E_S(0)$ and χ are the surface energy of the spherical liquid drop and fissility parameter, respectively.

$$\chi = \frac{3}{5}\frac{e^2}{r_0}\frac{Z^2/A}{2a_2\{1-\kappa[(N-Z)/A]^2\}} \qquad (3)$$

The fissility parameter, and the values of the coefficients a_2 and κ are crucial a determining the shape dependence of the liquid drop energy of fission barriers [2].

In order to account for the average properties of a single- particle spectrum, ones introduces a smooth function $\tilde{g}(\in,\beta)$ [2]. The auxiliary function,

$$\tilde{G}(\varepsilon,\beta) = (\pi^{1/2}\tilde{\gamma})\sum_{\nu}\exp(-\left\{\frac{\varepsilon-\varepsilon_\nu(\beta)}{\tilde{\gamma}}\right\}^2) \qquad (4)$$

The averaging interval $\tilde{\gamma}$,

$$\tilde{\gamma} \approx \hbar w_0 \approx 7 - 10 MeV \qquad (5)$$

The function $\tilde{G}(\varepsilon,\beta)$ is obtained by smearing out the single particle energies ε_ν, over an energy range of the order of $\hbar w_0$ [2].

The uniform level density can be written as

$$\tilde{g}(\varepsilon,\beta) = \tilde{G} - \frac{1}{4}\tilde{\gamma}^2\left(\frac{\partial^2\tilde{G}}{\partial\varepsilon^2}\right) + ... + a_{2m}\tilde{\gamma}^{2m}\left(\frac{\partial^{2m}\tilde{G}}{\partial\varepsilon^{2m}}\right) \qquad (6)$$

where the smoothing function,

$$\xi(x) = (\pi)^{-1/2} \exp(-x^2) \sum_{k=0,2}^{2m} a_k H_k(x) \tag{7}$$

$H_k(x)$ is the Hermite polynomials, and the coefficients a_k are given by the recurrence relation.

It is convenient to introduce another density function, $g_{sh}(\varepsilon, \beta)$ in order to describe the local level density of the single-particle spectrum. The variations in the single-particle level density caused by the shells can be described by the function [2]

$$\delta g(\varepsilon, \beta) = g_{sh}(\varepsilon, \beta) - \tilde{g}(\varepsilon, \beta) \tag{8}$$

The sum of single-particle energies are given by

$$U = 2 \int_{-\infty}^{\lambda_{sh}(\beta)} \varepsilon g_{sh}(\varepsilon, \beta) d\varepsilon \tag{9}$$

The energy shell corrections can be written as

$$E_{shell} = 2\left[\int_{-\infty}^{\lambda_{sh}(\beta)} \varepsilon g_{sh}(\varepsilon,\beta)d\varepsilon - \int_{-\infty}^{\tilde{\lambda}(\beta)} \varepsilon \tilde{g}_{sh}(\varepsilon,\beta)d\varepsilon\right] \tag{10}$$

Renormalization in BCS theory

In this work, the pairing correlation energy was evaluated as in the commonly used prescriptions of the BCS approach. We were also calculated of the pairing energies [6]. In this way, we valued only the essential energy variation gap Δ and the pairing energy due to the shell structure [2].

We start with the following BCS equation:

$$\frac{2}{G} = \int_{\tilde{\lambda}-\Omega}^{\tilde{\lambda}+\Omega} \frac{\tilde{g}(E)dE}{\left[\left(E-\tilde{\lambda}\right)+\tilde{\Delta}^2\right]^{-1/2}} \approx 2\tilde{g}(\tilde{\lambda})\ln\left(\frac{2\Omega}{\tilde{\Delta}}\right) \tag{11}$$

where, Ω is the cutoff energy and $\tilde{g}(\tilde{\lambda})$ is the average level density at the Fermi energy.

We define the energy P of the pairing correlations as the difference between the sums of single-particle energies evaluated with and without the pairing correlations [2].

$$P = \Sigma(\varepsilon_\nu - \lambda)\text{sign}\left[\varepsilon_\nu - \lambda_0\right] - \frac{(\varepsilon_\nu - \lambda)^2 + \frac{1}{2}\Delta^2}{\left[(\varepsilon_\nu - \lambda)^2 + \Delta^2\right]^{1/2}} \tag{12}$$

The shell correction in the pairing energy is defined as

$$E_{pair} = P - \tilde{P} \tag{13}$$

Where $\tilde{P}$ is the pairing correlation energy fort he uniform distribution

$$\tilde{P} = -\frac{1}{2}\tilde{g}(\tilde{\lambda})\tilde{\Delta}^2 \tag{14}$$

$$E_{Barrier} = E_{LDM} + E_{Shell} \tag{15}$$

III. RESULTS

Using the Strutinsky method with the Pashkevich parametrizations of nuclear shape, as introduced in the BARRIER code [2], we have carried out fission barrier and deformation. The parameter ε is associated with elongation and defines a concrete Cssini ovaloid, whereas α_4 is connected to the deformation parameter of hexadecapolar momentum, and are coefficients of Legendre polynomials series expansion. In Fig. 1-5 we showed our calculated fission barriers as functions of ε and α_4.

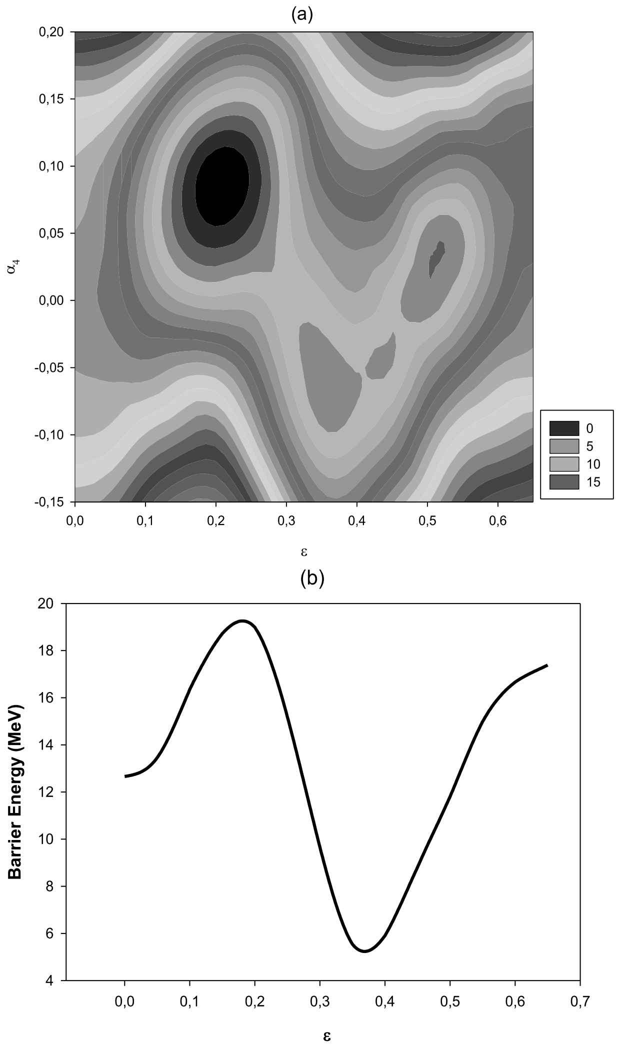

Fig.1: (a) Calculated fission barrier in the (ε, α_4) plane for ^{230}Pa. The contour line separation is 0,05 MeV. (b) Fission path through the barrier.

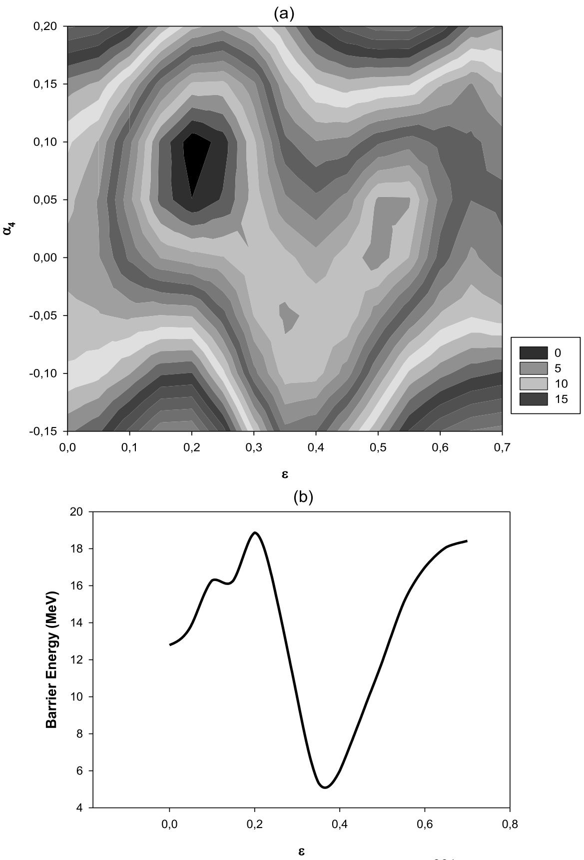

Fig.2 The same as in fig. 1 but for ^{231}Pa.

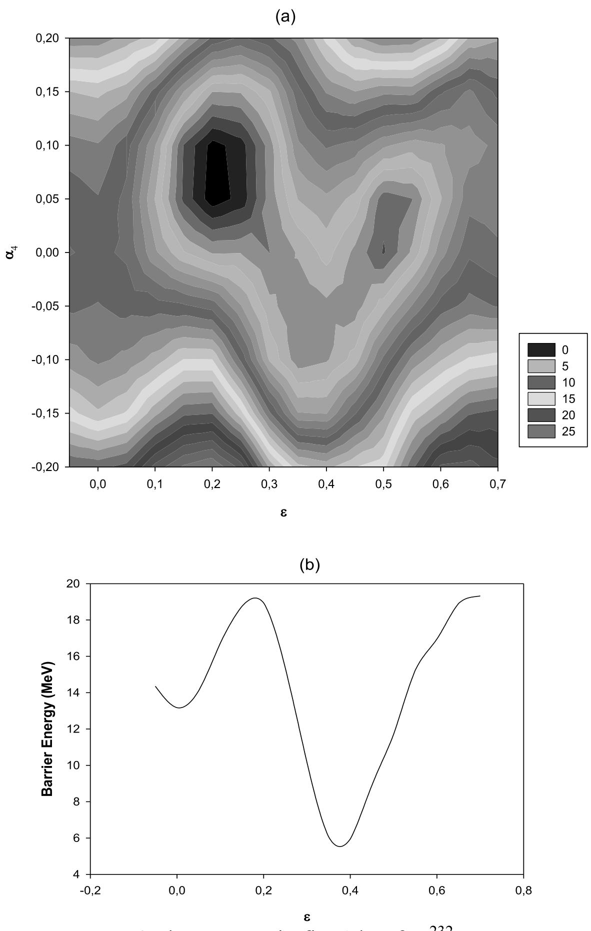

Fig.3 The same as in fig. 1 but for ^{232} Pa.

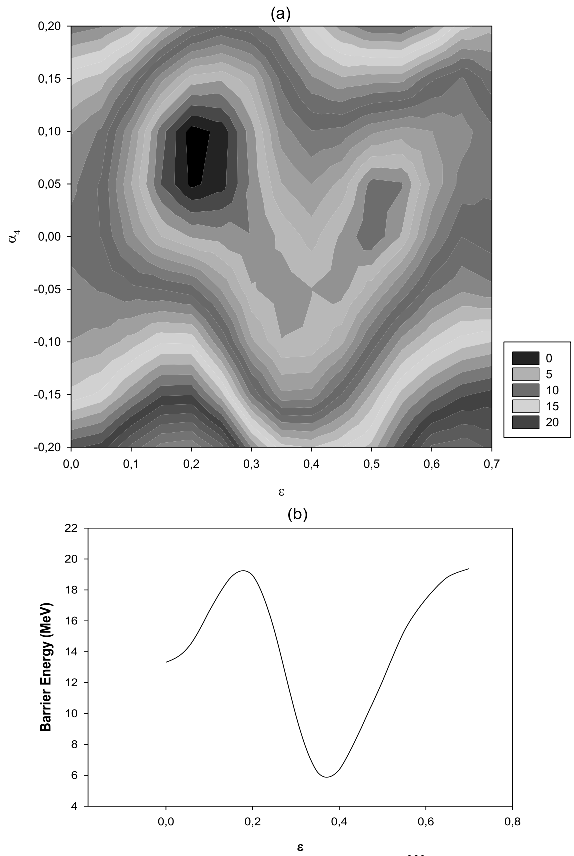

Fig.4 The same as in fig 1 but for ^{233} Pa.

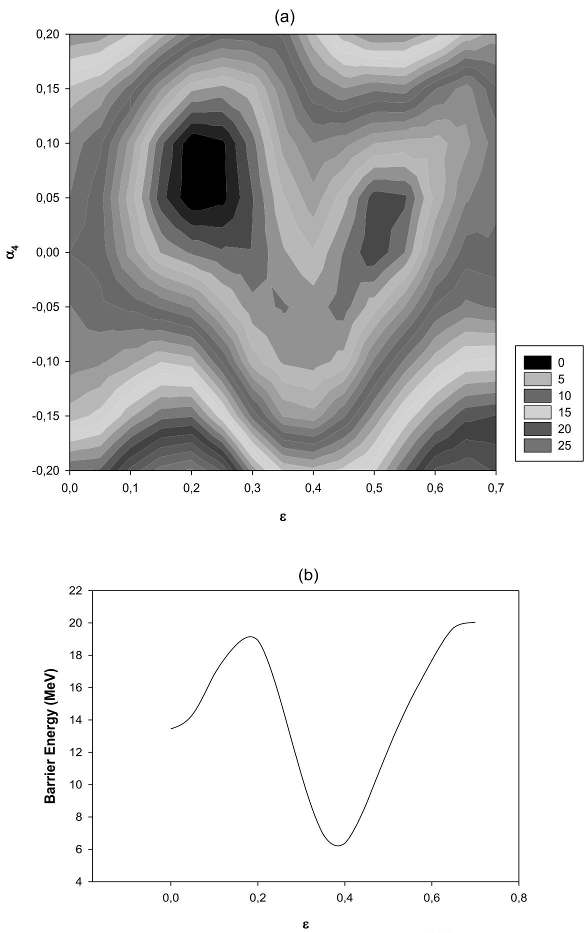

Fig.5 The same as in fig 1 but for ^{234} Pa

As shown, the deformation coordinates are sensitive to the adopted in $^{230-234}$ Pa, which only changes the relative height of the fission barrier. The fission barrier is composed of the liquid drop energy and the shell correction energy.

Table1. Results of Fission barrier

	Fission Barrier		
	This work	*Experimental*	*ETFSI*
^{230}Pa	5,90	5,80	6,00
^{231}Pa	5,61	5,50	5,80
^{232}Pa	6,06	6,40	5,90
^{233}Pa	5,92	5,80	6,00
^{234}Pa	6,18	6,15	5,30

Table 2. Deformation Parameters

	First Minimum		*Second Minimum*	
	ε	α_4	ε	α_4
^{230}Pa	0,215	0,100	0,488	0,0
^{231}Pa	0,209	0,100	0,495	0,0
^{232}Pa	0,210	0,100	0,496	0,0
^{233}Pa	0,212	0,100	0,497	0,0
^{234}Pa	0,211	0,100	0,493	0,0

The fission barriers for Pa isotopes were calculated by using the BARRIER code [2]. In Table 1, The values obtained were compared to the experimental results [7] and ETFSI results [8]. As we can see, the BARRIER code fission barriers are in good agreement with the experimental values.

By using the above mentioned method, deformations corresponding to the second minimum could be obtained. In Table 2 we showed the deformation parameters for the ground state and the second minimum of $^{230-234}$Pa. The parameters used as starting values were taken from Chepurnov[9]. Both the ε and α_4 deformation parameters in Pa isotopes are very similar around the first and second minimum.

IV. CONCLUSIONS

We have calculated the details the fission barriers of $^{230-234}$Pa. Shell effects should be present in these nuclei, come into existence structures connected to the barrier energy, saddle point, first and second minima. Shell effects are most probably characteristics responsible for fission decay strange

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