BALKAN PHYSICS LETTERS

31 January 2011
BPL, 19, 191009 , pp. 85 – 94 , (2011)

AXIALLY DEFORMED RELATIVISTIC MEAN FIELD CALCULATIONS ON THE PROPERTIES OF ISOTOPIC CHAIN OF SUPER-HEAVY Hs NUCLEI

A. H. YILMAZ
Physics Department, Karadeniz Technical University,
Trabzon, TURKEY.
and
T. BAYRAM
Physics Department, Karadeniz Technical University,
Trabzon, TURKEY.
&
Physics Department, Sinop University,
Sinop, TURKEY.

Abstract. At present many laboratories in the field of nuclear physics study on the Super-Heavy Nuclei (SHN) because the successful synthesizing of super-heavy elements (SHE) in lab has stimulated the research. This speeds up the researches on SHN theoretically. There are some theoretical studies on the Super-Heavy Nuclei based on the self-consistent mean field models. In this study, the structures of the Super-Heavy Hs nuclei for a wide range of neutron numbers were investigated using the deformed relativistic mean field (RMF) theory with new Lagrangian parameters set. Binding energy, quadrupole moment, deformation parameter, neutron radii, proton radii, two-neutron separation energy, α-decay energy and α-decay half-lives of Hs isotopes were calculated. The results were compared with available experimental data and predictions of some nuclear models. The results show that RMF theory with newly revised NL3* parameters set yields successful description for ground-state properties of isotopic chains of super-heavy Hs nuclei.

PACS numbers: 21.10.Dr, 21.10.Jv, 21.10.Ky, 21.10.Tg, 23.60.+e, 27.90.+b

Keywords
Relativistic mean field, even-even Hs nuclei, binding energy, alpha decay properties, deformations

INTRODUCTION

Investigation of several super-heavy isotopes beyond Fermium (Z=100) has received much attention because of the success of syntheses of Super-Heavy Nuclei (SHN) in lab [1-11]. The investigations of the properties of these nuclei are very important for good understanding some features of nuclear structure such as deformations, binding energy, α-decay energy, α-halflives in super-heavy region. There are some theoretical studies on SHN on the macroscopic-microscopic mass models or self-consistent mean field models [12-22]. Hassium (Hs, Z=108) considered here was first synthesized in ${}^{208}_{82}Pb + {}^{58}_{26}Fe \rightarrow {}^{265}_{108}Hs + n$ reaction in 1984 by a German research group in Darmstadt. After this discovery, in chronological order ^{264}Hs, ^{267}Hs, ^{269}Hs, ^{266}Hs, ^{275}Hs, ^{270}Hs, ^{271}Hs, ^{263}Hs and ^{268}Hs were discovered in various reactions. In this study, we investigated ground-state properties of some even-even Hs nuclei (from A=250 to A=276) within the axially deformed Relativistic Mean Field (RMF) theory with newly revised NL3* parameters set because RMF models with 9 parameters have reached a high degree of accuracy in the description of nuclear ground-state observables [23-24].

RELATIVISTIC MEAN FIELD THEORY

In the RMF theory, nucleons are described as Dirac particles moving in meson fields. These nucleons interact with each other via the exchange of the scalar mesons σ, iso-vector mesons ρ and vector mesons ω, and also the photons (A). The starting point of RMF theory is an effective Lagrangian density includes these interactions is given as

$$\begin{aligned}\mathcal{L} = {} & \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - M\right)\psi + \tfrac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma - U(\sigma) - g_{\sigma}\bar{\psi}\sigma\psi \\ & - \tfrac{1}{4}\Omega^{\mu\upsilon}\Omega_{\mu\upsilon} + \tfrac{1}{2}m_{\omega}^{2}\omega^{\mu}\omega_{\mu} - g_{\omega}\bar{\psi}\gamma^{\mu}\psi\omega_{\mu} \\ & - \tfrac{1}{4}\vec{R}^{\mu\upsilon}\vec{R}_{\mu\upsilon} + \tfrac{1}{2}m_{\rho}^{2}\vec{\rho}^{\mu}\vec{\rho}_{\mu} - g_{\rho}\bar{\psi}\gamma^{\mu}\vec{\tau}\psi\vec{\rho}_{\mu} \\ & - \tfrac{1}{4}F^{\mu\upsilon}F_{\mu\upsilon} - e\bar{\psi}\gamma^{\mu}\tfrac{1-\tau_{3}}{2}A_{\mu}\psi,\end{aligned} \tag{1}$$

where Dirac spinor ψ denotes the nucleon with mass M. The isoscalar scalar σ-meson and isoscalar vector ω-meson yields medium-range attractive and short-range repulsive interactions, respectively. The isovector vector ρ-meson provides the isospin asymmetry. Masses of these mesons are denoted by m_{σ}, m_{ω} and m_{ρ}. g_{σ}, g_{ω} and g_{ρ} correspond to the nucleon-meson coupling constants. $U(\sigma\,)$ term includes nonlinear isoscalar scalar terms need to be introduced for a quantitative description of nuclei and nuclear matter and its explicit form is $U(\sigma\,) = \frac{1}{2}m_{\sigma}^{2}\sigma^{2} + \frac{1}{3}g_{3}\sigma^{3} + \frac{1}{4}g_{3}\sigma^{4}$. τ is the isospin of the nucleon and τ_{3} is its third component. The field tensors of the vector mesons and electromagnetic field take the forms:

$$\Omega^{\mu\upsilon} = \partial^{\mu}\omega^{\upsilon} - \partial^{\upsilon}\omega^{\mu},$$
$$\vec{R}^{\mu\upsilon} = \partial^{\mu}\vec{\rho}^{\upsilon} - \partial^{\upsilon}\vec{\rho}^{\mu}, \qquad (2)$$
$$F^{\mu\upsilon} = \partial^{\mu}A^{\upsilon} - \partial^{\upsilon}A^{\mu}.$$

The classical variational principle gives the equations of motion. At this stage, fields are taken as the c-number or classical fields. This results into a set of coupled equations namely the Dirac equation with potential terms for the nucleons and the Klein-Gordon equations with sources for the mesons and the photon. Dirac equation for the nucleon is

$$\{-i\boldsymbol{\alpha}\boldsymbol{\nabla} + V(\boldsymbol{r}) + \beta[M + S(\boldsymbol{r})\}\psi_i = \varepsilon_i\psi_i\,, \qquad (3)$$

where $V(\mathbf{r})$ denotes the vector potential: $V(\mathbf{r}) = g_{\omega}\omega_0(\mathbf{r}) + g_{\rho}\tau_3\rho_0(\mathbf{r}) + e\frac{1+\tau_3}{2}A_0(\mathbf{r})$ and $S(\mathbf{r})$ is the scalar potential: $S(\boldsymbol{r}) = g_{\sigma}\sigma(\boldsymbol{r})$ the latter contributes to the effective mass as: $M^{*} = M + S(r)$.

The Klein-Gordon equations for the meson and electromagnetic fields with the nucleon densities as sources are

$$\{-\Delta + m_{\sigma}^2\}\sigma(r) = -g_{\sigma}\rho_s(r) - g_2\sigma^2(r) - g_3\sigma^3(r)$$
$$\{-\Delta + m_{\omega}^2\}\omega_0(r) = g_3\rho_v(r)$$
$$\{-\Delta + m_{\rho}^2\}\rho_0(r) = g_{\rho}\rho_3(r) \qquad (4)$$
$$-\Delta A_0(r) = e\rho_c(r).$$

The corresponding source terms for the mesons and photons are

$$\rho_s = \sum_{i=1}^{A} n_i\bar{\psi}_i\psi_i$$
$$\rho_v = \sum_{i=1}^{A} n_i\psi_i^{\dagger}\psi_i$$
$$\rho_3 = \sum_{i=1}^{Z} n_i\psi_p^{\dagger}\psi_p - \sum_{i=1}^{N} n_i\psi_n^{\dagger}\psi_n \qquad (5)$$
$$\rho_c = \sum_{p=1}^{Z} n_i\psi_p^{\dagger}\psi_p.$$

Here the summations are taken over the valence nucleons only, i.e., no-sea approximation is adopted. The coupled equations (3) and (4) can be self-consistently solved by iteration using the mean field approximation. The occupation number n_i is introduced to account for pairing which is important for open shell nuclei. In the presence of pairing the partial occupancies n_i are obtained in the constant gap approximation (BCS) through the well known expression:

$$n_i = \frac{1}{2}\left(1 - \frac{\varepsilon_i - \lambda}{\sqrt{(\varepsilon_i - \lambda)^2 + \Delta^2}}\right) \quad (6)$$

where ε_i and λ is the single-particle energy for the state i and chemical potential for protons or neutrons, respectively. Details can be found in Ref. [25].

For axially deformed nuclei, it is useful to work with cylindrical coordinates: $x = r_\perp cos\varphi$, $y = r_\perp sin\varphi$ and z. The Dirac spinor ψ_i is characterized by the quantum numbers Ω_i (the eigenvalue of the angular momentum operator j_z), π_i (parity) and t_i (isospin) and it has the form

$$\psi_i = \begin{pmatrix} f_i(r) \\ ig_i(r) \end{pmatrix} = \frac{1}{\sqrt{2\pi}} \begin{pmatrix} f_i^+(z, r_\perp)e^{i(\Omega_i - 1/2)\varphi} \\ f_i^-(z, r_\perp)e^{i(\Omega_i + 1/2)\varphi} \\ ig_i^+(z, r_\perp)e^{i(\Omega_i - 1/2)\varphi} \\ ig_i^-(z, r_\perp)e^{i(\Omega_i + 1/2)\varphi} \end{pmatrix} \chi_{t_i}(t), \quad (7)$$

The densities can be represented as

$$\rho_{s,v} = 2\sum_{i>0} n_i \left[(|f_i^+|^2 + |f_i^-|^2) \mp (|g_i^+|^2 + |g_i^-|^2)\right], \quad (8)$$

and, in a similar way, ρ_3 and ρ_c because the contributions to the densities of the time reversal states, i and $\bar{\imath}$, are identical for nuclei with time reversal symmetry. The sum here runs over only states with positive Ω_i. For solutions of the equations (3) and (4), the spinors $f_i^\pm(z, r_\perp)$ and $g_i^\pm(z, r_\perp)$ can be expanded in terms of the eigenfunctions of a deformed axially symmetric oscillator potential and the solution of the problem is transformed into a diagonalization of a Hermitian matrix. Details can be found in Ref. [26].

The total energy of the system is:

$$E_{RMF} = E_n + E_\sigma + E_\omega + E_\rho + E_c + E_{cm} + E_{pair} + E_{nl} - AM \quad (9)$$

where E_n is the sum of the energy for nucleon ε_i, E_σ, E_ω, E_ρ are the contributions of the meson fields, E_c, E_{cm}, E_{pair}, E_{nl} and AM is the contributions of the Coulomb field, center of mass correction, pairing, non-linear part and total mass, respectively.

DETAILS OF CALCULATIONS

The Hs isotopes considered in this study were even-mass nuclei with mass number A = 250 up to 276. Equation of motions of both nucleons and mesons in the expansion method with the axially deformed harmonic-oscillator basis were solved. In order to get a better computational result, the numbers of shells taken into account were 20 for both the fermionic and bosonic expansion. Pairing was considered by the constant gap approximation (BCS). The pairing gap for neutron and proton were chosen as $\Delta_n = \Delta_p = 11.2/\sqrt{A}$ for an even number of nucleons. The basis parameters $\hbar\omega$ used for the calculations was taken to be $41A^{-1/3}$. The convergence of the numerical calculation on binding energy and deformation was very good. For all nuclei calculated in this work, initial deformation parameter β_0 was chosen as 0.3 which is reliable for these calculations. Well known, different choices of β_0 are not effect on convergence of deformations. It leads to different iteration numbers of the self-consistent calculation and different computational time. But physical quantities such as the binding energy and the deformation change very little. There are many parameters sets for RMF calculations which provide nearly equal quality of description for stable nuclei. In this study we used newly revised NL3* [27] parameters set for all calculations. Also, for comparison of binding energies NL-SH [28] parameters set was used. NL3* and NL-SH parameters sets were listed in Table 1.

Table 1: NL3* and NLSH parameters sets for RMF calculations

Parameter	NLSH	NL3*
M (MeV)	939.000	939.000
m_σ (MeV)	526.059	502.574
m_ω (MeV)	783.000	782.600
m_ρ (MeV)	763.000	763.000
g_σ	10.4440	10.0944
g_ω	12.9450	12.8065
g_ρ	4.3830	4.5748
g_2 (fm^{-1})	-6.9093	-10.8093
g_3	-15.8337	-30.1486

RESULTS AND DISCUSSION

The calculated and experimental total binding energies of Hs isotopes with mass number are given in Fig. 1. We calculated total binding energy for Z=108 isotopes in RMF theory with NL3* [27] and NLSH [28] parameters sets. Also, Fig. 1, presents the experimental data taken from [29] and predictions of Finite Range Droplet Model (FRDM) taken from [30] for comparison. As can be seen in Fig. 1, the predictions of RMF theory with NL3* parameters set and predictions of FRDM are in good agreement with experimental data. Although the general trend of the curve of the RMF theory with NLSH force is compatible with the experimental data, as seen in Fig. 1 the predictions of RMF theory with NL3* force more reliable. Because of this reason we used NL3* parameters set for calculation of other ground state properties of Hs isotopes. Maximum deviation of binding energy of Hs isotopes between RMF with NL3* and experimental data at 268 mass number and its value 12.328 MeV.

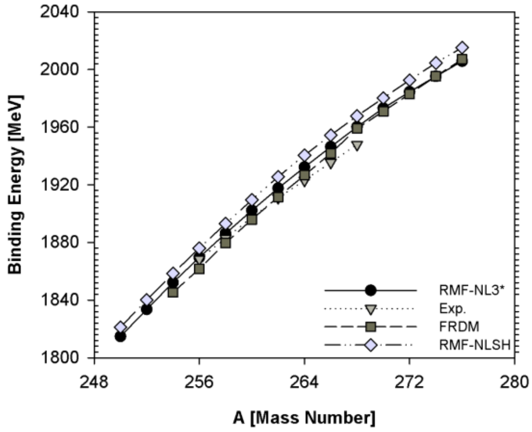

Figure 1: Comparison of theoretical and experimental total binding energies for the Hs isotopic chain.

The calculated ground-state quadrupole deformation parameters β_2 in this study and the deformations calculated with both FRDM and Extended Thomas Fermi Model with Strutinski Integral (ETF-SI) taken from Ref. [30] and [31], respectively are shown in Fig. 2. As can be seen in Fig. 2, Hs isotopes calculated with RMF-NL3* are well deformed, and with adding neutron the ground state shape has no sudden change. The deformation magnitudes and tendency, except N=166 and 168, are in coincidence with the predictions of the FRDM and ETF-SI. The curve of RMF calculation showed the descent of the deformation from N=162 as we expected, β_2=0.26, this shows deformed sub-closure. Deformations of Hs isotopes could be extended N>168 region for Hs isotopes.

Well known, there exist many experimental indications showing that atomic nuclei possess a shell-structure and that they can be constructed, like atoms, by filling successive shells of an effective potential well. In order to get further information on shell effects, the two-neutron separation energies by RMF with NL3* parameters set for Hs isotopes were plotted as a function of neutron number. Also experimental data [29] and predictions of FRDM [30] are shown for comparison. The two-neutron separation energies S_{2n} were derived from the binding energies of the two neighboring even isotopes using $S_{2n} = B(Z,N) - B(Z,N-2)$ equation. It can be seen in Fig. 3, the two-neutron separation energies calculated in our study are good agreement with available experimental data rather than predictions of FRDM. A king in the S_{2n} values is visible at N=162. This reveals the sub closure shell.

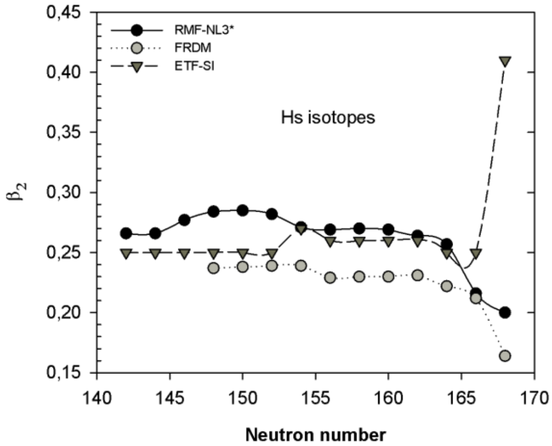

Figure 2: The ground-state quadrupole deformation parameter β_2.

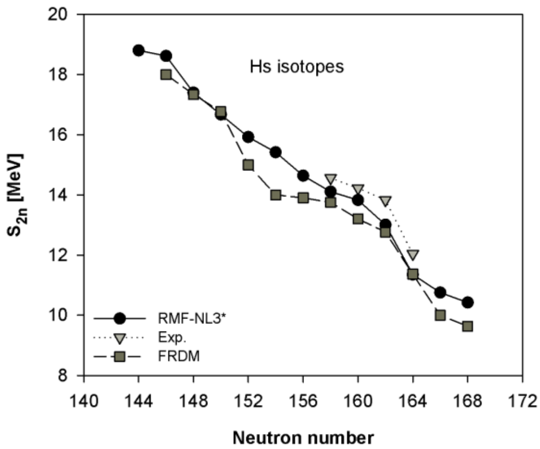

Figure 3: The two-neutron separation energies for Hs isotopes

In Fig. 4a, calculated α-decay energies (Qα) of Hs isotopes are shown. The $Q\alpha$ energy is obtained from the relation $Q\alpha(N,Z) = \mathrm{BE}(N,Z) - \mathrm{BE}(N-2,Z-2) - \mathrm{BE}(2,2)$. Here, $\mathrm{BE}(N,Z)$ is the binding energy of the parent nucleus with neutron number N and proton number Z, BE(2,2) is the binding energy of the α particle chosen as 28.296 MeV and $\mathrm{BE}(N-2,Z-2)$ is the binding energy of the daughter nucleus after the emission of an α particle. The binding energies of the parent (Hs) and daughter nuclei (Sg) are obtained by using the RMF formalisms. It can be seen in Fig. 4, the result of our calculations are agreement with experimental data. Also, $\log_{10}T\alpha(s)$ values corresponding to halflives of Hs isotopes were estimated using phenomenological Viola and Seaborg [32] formula: $log_{10}T_{\alpha}(s) = \frac{aZ-b}{\sqrt{Q_\alpha}} - (cZ+d)$, where Z is the atomic number of the parent nucleus, a=1.661175, b=8.5166, c=0.20228 and d=33.9069. The calculated $\log_{10}T\alpha(s)$ in our study for Hs isotopes as a function of neutron number are shown in Fig. 4b. Also the estimates of $\log_{10}T\alpha(s)$ for Hs isotopes derived from experimental data using Viola and Seaborg formula are shown for comparison. The calculated $T\alpha$ values, the other calculated ground state properties which are total binding energy, neutron radii, proton radii, total quadrupole moments and α-decay energies were listed in Table. 2.

Table 2: The calculated some ground-state properties of Hs isotopes.

Nuclei	BE [MeV]	R_n [fm]	R_p [fm]	Q_T [barn]	Q_α (theor.) [MeV]	Q_α (exp.) [MeV]	T_α [theor.]
^{250}Hs	1814.750	6.093	5.992	905.759	11.428		6.554 μs
^{252}Hs	1833.552	6.116	6.002	920.067	10.982		68.104 μs
^{254}Hs	1852.168	6.144	6.019	969.703	10.225		5.113 ms
^{256}Hs	1869.568	6.171	6.033	1008.142	10.206		5.733 ms
^{258}Hs	1886.238	6.194	6.044	1025.268	10.528		0.855 ms
^{260}Hs	1902.160	6.215	6.054	1026.594	10.612		0.531 ms
^{262}Hs	1917.578	6.235	6.062	999.957	10.623		0.498 ms
^{264}Hs	1932.216	6.258	6.074	1004.403	10.364	10.59	2.233 ms
^{266}Hs	1946.322	6.284	6.087	1020.093	10.106	11.00	10.532 ms
^{268}Hs	1960.152	6.308	6.099	1029.195	9.713	9.90	125.783 ms
^{270}Hs	1973.160	6.330	6.108	1024.508	9.645	9.30	196.176 ms
^{272}Hs	1984.512	6.352	6.118	1009.632	10.244	10.10	4.561 ms
^{274}Hs	1995.268	6.358	6.115	858.360	9.695	9.50	141.423 ms
^{276}Hs	2005.692	6.376	6.122	803.945	8.678	8.80	189.920 ms

BE, total binding energy; R_n, neutron radii; R_p, proton radii; Q_T, total quadrupole moment; Q_α, α-decay energy; T_α, α-decay half-life

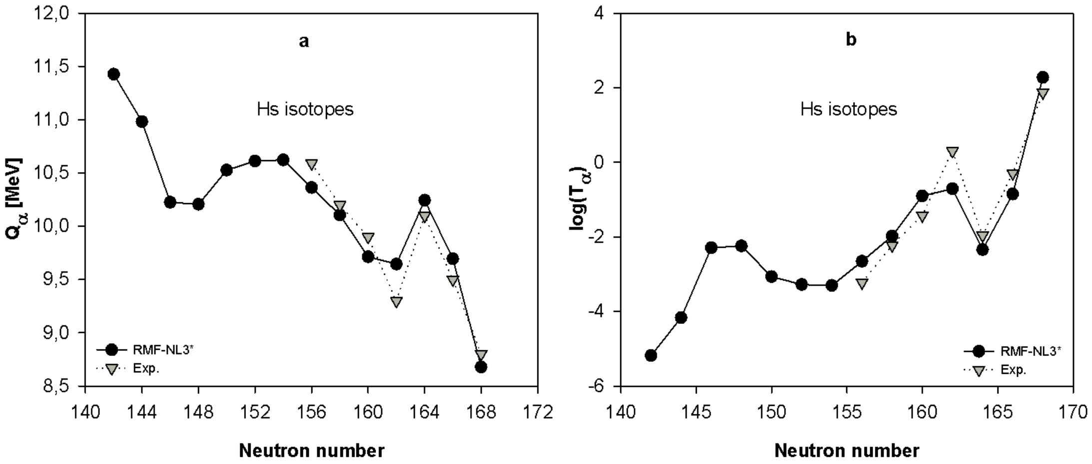

Figure 4: The α-decay energies (a) and $\log_{10}T(s)$ (b) for Hs isotopes as a function of neutron number.

CONCLUSIONS

The ground state nuclear properties of even-even Hs isotopes which has 108 protons were investigated using Relativistic Mean Field (RMF) Theory with NL3* parameters set. The NL3* parameters set revised last year provides good description for binding energies of Hs isotopes rather than NLSH parameters set. Our calculation on two-neutron separation energies and α-decay energies for even-even Hs isotopes were in good agreement with available experimental data. Also, the $\log_{10}T\alpha(s)$ values of Hs isotopes were estimated using the results of the RMF theory in Viola and Seaborg formula. Almost the results were in agreement with available experimental results. Beside these, neutron radii, proton radii and total quadrupole moments of Hs isotopes were calculated for further information. All of these reasons mention above it can be concluded that RMF theory with NL3* parameters set provides successful description of ground-state nuclear properties of Hs isotopes.

REFERENCES

[1] R. C. Barber et al., Pure Appl. Chem. **63** No:6, 879 1991.
[2] S. Hofmann et al., Z. Phys. A. **350**, 277 1995.
[3] S. Hofmann et al., Z. Phys. A. **350**, 281 1995.
[4] S. Hofmann et al., Z. Phys. A., **354**, 229 1996.
[5] S. Hofmann, Rep. Prog. Phys. **61**, 636 1998.
[6] Yu. Ts. Oganessian et al., Nature. **400**, 242 1999.
[7] Yu. Ts. Oganessian et al., Phys. Rev. Lett. **83**, 31 1999.

[8] S. Hofmann and G. Münzenberg., Rev. Mod. Phys. **72**, 733 2000.
[9] Yu. Ts. Oganessian, Rad. Phys. Chem. **61**, 259 2001.
[10] F. P. Hehenberger, Nucl. Phys. A. **719**, 185c 2003.
[11] Yu. Ts. Oganessian, Nucl. Phys. A. **834**, 331c 2010.
[12] S. Cwiok, W. Nazarewicz and P. H. Heenen, Phys. Rev. Lett. **83**, 1108 1999.
[13] W. D. Myers and W. J. Swiatecki, Phys. Rev. C. 58, 44610 2000.
[14] Z. Ren and H. Toki, Nucl. Phys. A. **689**, 691 2001.
[15] K. Rutz et al., Phys. Rev. C. **56**, 238 1997.
[16] Z. Ren et al., J. Phys. G. **22**, 1739 1996.
[17] M. Bender et al., Phys. Rev. C. **60**, 34304 1999.
[18] Z. Ren, J. Nucl. Radiochem. Sci, **3** No:1, 195 2002.
[19] S. Gmuca, Acta Phys. Hung. A. **19**/1-2, 155 2004.
[20] H. F. Zhang et al., Int. J. Mod. Phys. E. **15** No: 7, 1613 2006.
[21] L. Ma et al., Int. J. Mod. Phys. E. **17**, Supplement, 97 2008.
[22] F. Zhou and J. Y. Guo, Int. J. Mod. Phys. E. **17** No: 7, 1309 2008.
[23] P. G. Reinhard, Rep. Prog. Phys. **52**, 439 1989.
[24] M. Bender, P. H. Heenen and P. G. Reinhard, Rev. Mod. Phys. **75**, 121 2003.
[25] P. Ring, Prog. Part. Nucl. Phys. **37**, 193 1996.
[26] P. Ring, Y. K. Gambhir and G. A. Lalazissis, Comp. Phys. Commun **105**, 77 1997.
[27] G. A. Lalazissis et al., Phys. Lett. B **671**, 36 2009.
[28] M. M. Sharma and M. A. Nagarajan, Phys.Lett. B **312**, 377 1993.
[29] G. Audi, A. H. Wapstra and C. Thibault, Nucl. Phys. A **729**, 337 2003.
[30] P. Möller, R. Nix, W. D. Myers and W. J. Swiatecki, At. Data Nucl. Data Tables. **59**, 185 1995.
[31] Y. Aboussir, J. M. Pearson, A. K. Dutta and F. Tondeur, At. Data Nucl. Data Tables, **61**, 127 1995.
[32] V. E. Viola, Jr. and G. T. Seaborg, J. Inorg, Nucl. Chem. **28**, 741 1966.