TECHNOLOGY-ENHANCED MATHEMATICAL MODELING IN FLIPPED LEARNING ENVIRONMENTS: CHALLENGES FOR PRE-SERVICE TEACHERS

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Abstract

Due to the sudden closure of the university following a major earthquake, a mathematics education course was rapidly transformed into a fully online, technology-enhanced learning environment, offering a unique opportunity to examine how pre-service middle school mathematics teachers engage with mathematical modeling tasks within a flipped learning framework. Adopting a phenomenological approach, the research involves 51 pre-service middle school mathematics teachers enrolled in a mathematical modeling course in mathematics education. Data were collected through assignments, videos, reports from the Academic Learning Management System, GeoGebra records, and semi-structured interviews. The data obtained from the Academic Learning Management System and GeoGebra were analyzed via descriptive analysis, whereas the semi-structured interviews were examined via content analysis. Pre-service middle school mathematics teachers experienced difficulties in modeling competencies, particularly in applying subject matter knowledge effectively and integrating technology into the modeling process. Technology-enhanced learning environments emerged as a critical support mechanism for fostering the use of content knowledge and promoting mathematical modeling as a problem-solving approach. Structured guidance and the integration of interactive digital tools within flipped learning environments proved to be effective in addressing the challenges encountered during the modeling tasks.
Keywords: *flipped learning, mathematical modeling, online learning, pre-service mathematics teachers*

Introduction

In today's educational landscape, the increasing frequency of unexpected events has accelerated the integration of technological innovations in mathematics education. To preserve educational continuity, educators have been compelled to adopt flexible models and alternative learning environments in university courses (Jummai, 2021; Viju, 2021). Consequently, the functional use of technology in course design has become indispensable (Al-Khasawneh & Obeidallah, 2015; Jung & Lee, 2019; Okai et al., 2014). One prominent response to these changing conditions is the flipped learning model (FLM), which transforms traditional teaching and learning methods (Bergmann & Sams, 2012).

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The FLM enables students to access theoretical content outside class through videos and digital materials, allowing in-class time to focus on interaction, creativity, and discussion (Bergmann & Sams, 2012; Bui & Nguyen, 2023; Truong & Diep, 2023). Its capacity to integrate various technologies enhances its instructional impact (Kozikoğlu, 2019). This approach supports student autonomy, enabling them to learn at their own pace, take responsibility for learning, and develop independent learning skills (Effendi et al., 2024; Fulton, 2012; Yang & Chen, 2020). As a result, engagement and motivation increase (O'Flaherty & Phillips, 2015), fostering more productive learning environments (Strayer, 2012).

This study examines the abrupt transformation of a mathematical modeling course, initially designed with FLM, into an online format due to the earthquake in Türkiye. Such emergencies necessitate swift shifts from traditional models, compelling educators to restructure instruction with flexible and tech-integrated approaches (Effendi et al., 2024). The FLM, in this context, enhances students' individual learning responsibility, supports the integration of in- and out-of-class activities, and proves effective for online learning. Especially in problem-solving-oriented courses such as mathematical modeling, students grasp core concepts outside class and reinforce them through active engagement during sessions, making FLM a viable model in online settings.

Mathematical modeling is critical in helping students make sense of mathematics and relate it to real life (Blum & Leiβ, 2007). Modeling is a powerful teaching tool that allows mathematical concepts to be associated with real-world problems by moving beyond the abstract plane (Wilkins, 2010). In this process, the use of technology, in particular, enables students to develop more complex models, switch between different representations, and gain a deeper understanding of these representations (Siller & Greefrath, 2010). Such course models, where technology is actively used, allow students to access theoretical information offline and use class time for more creative and problem-solving-focused activities.

Despite the increasing emphasis on mathematical modeling in teacher education, little is known about how pre-service middle school mathematics teachers (PSTs) engage with this process in technology-supported, flipped learning environments.

Flipped Learning and Its Use in Mathematics Education

The FLM has gained significant momentum in mathematics education because of its potential to increase student engagement and learning outcomes. This model enhances student engagement, comprehension, and academic performance while also fostering collaboration, autonomy, self-regulation, and positive attitudes toward mathematics (Abeysekera & Dawson, 2015; Fernández-Martín et al., 2020; Kiem & Keodavan, 2024). In addition, technology-enabled course materials help students visualize challenging concepts and make abstract information more understandable (Chen et al., 2014).

While abstract mathematical concepts are often directly conveyed in traditional classroom settings, the FLM encourages active learning, allowing students to engage more deeply with mathematical concepts during face-to-face interactions with teachers and peers (Huesca et al., 2024; Khanova et al., 2015). Priyaadharshini and Sundaram (2018) have emphasized that active participation in learning activities before and during class improves students' critical thinking and problem-solving skills. The model also supports collaborative learning. Through discussion and group work, students reinforce their understanding and retention of mathematical ideas (Leary et al., 2021). Its interactive structure increases motivation (Love et al., 2014), fosters autonomy (Fernández-Martín et al., 2020), enhances learning outcomes (Brown et al., 2022), and promotes critical thinking (Pattimukay et al., 2023) and academic achievement (Sulaimon & Manditereza, 2024).

Despite its advantages, the implementation of FLM also presents challenges. Lundin et al. (2018) reported that some students are dissatisfied with the flipped classroom structure, emphasizing that the model requires careful planning and implementation to achieve educational goals effectively. In addition, the success of FLM largely depends on students' self-regulation and motivation, as students must take responsibility for their learning outside of the classroom (Yoon et al., 2020). Therefore, educators must provide adequate support and resources to help students successfully implement this model. High-quality materials and well-structured teacher guidance increase the contribution of this model to student success (Chen et al., 2014).

Overall, FLM emerges as an effective pedagogical approach for deepening conceptual understanding and improving mathematical problem-solving. Within this broader context, it offers significant promise for addressing instructional challenges in complex domains such as mathematical modeling.

Flipped Learning, Technology Integration, and Institutional Support in Mathematical Modeling

The mathematical modeling is a powerful tool that enables the analysis and prediction of complex systems (Wilkins, 2010). It enhances students' critical thinking skills and provides opportunities to relate mathematics to real-world problems (Blum & Niss, 1991). The modeling process involves multiple iterative stages, such as translating a real-life problem into a mathematical model, solving the model, and interpreting the results (Berry & Nyman, 1998; Haines & Crouch, 2007). This cyclical structure supports the development of problem-solving skills and fosters the perception of mathematics as a dynamic and applicable discipline (Borromeo Ferri, 2007).

The integration of technology has significantly enriched the modeling process by enabling visualization, exploration, and validation of models (Galbraith et al., 2007). Digital tools improve students' mathematical thinking and make modeling processes more interactive and effective (Greefrath et al., 2018; Hıdıroğlu, 2015). These tools help students bridge the gap between abstract mathematical concepts and practical applications, allowing them to verify and refine models through iterative processes (Jablonski et al., 2023). Research indicates that the incorporation of computational tools in classrooms deepens mathematical understanding and promotes the development of multiple mathematical connections (Silva et al., 2021).

Despite its advantages, mathematical modeling presents significant challenges for both students and educators. The abstract nature of mathematical concepts often makes it difficult for students to grasp key ideas (Blum & Niss, 1991). Additionally, transforming real-world problems into mathematical models requires a combination of mathematical knowledge, creativity, and critical thinking skills, which can be particularly demanding for learners (Hiebert & Grouws, 2007). Many PSTs may struggle with fully understanding mathematical concepts, which in turn reduces their confidence in teaching mathematical modeling (Blomhøj & Jensen, 2003). Furthermore, a lack of experience in planning and implementing modeling activities poses additional challenges for PSTs, making instructional design more complex (Frejd, 2012; Guerrero-Ortiz & Borromeo Ferri, 2022).

Another significant challenge is students' tendency to rely on familiar mathematical tools, such as continuous functions, even when alternative approaches like discrete modeling might be more suitable (Kaiser et al., 2011). This inclination can result in the superficial use of models and a reluctance to engage with more complex or irregular structures (Edgar, 2008). While some students demonstrate flexibility in adopting varied modeling approaches, many require explicit and structured support to broaden their strategy repertoire (Greefrath et al., 2022).

Flipped learning (FL) has emerged as an instructional approach that can address some of these challenges by restructuring traditional classroom teaching. By shifting theoretical content delivery to pre-class activities, FL frees up classroom time for active problem-solving, hands-on exploration, and collaborative engagement (Bergmann & Sams, 2012). Within this framework, explanatory videos (EVs) have emerged as crucial tools, providing structured and accessible learning resources that facilitate students' understanding of mathematical concepts (Cevikbas et al., 2025). FL has been shown to enhance student engagement, promote self-regulated learning, and support conceptual understanding, while its integration with mathematical modeling and real-world applications further strengthens students' mathematical reasoning and problem-solving abilities (Fernández-Martín et al., 2020; Kiem & Keodavan, 2024; Lee & Shin, 2022; Özen Ünal et al., 2023). In mathematical modeling, FL provides structured learning experiences where students first engage with theoretical content through online videos and then apply these concepts in in-class technology-supported activities, such as GeoGebra-based modeling tasks (Silva et al., 2021).

Nevertheless, implementing FL effectively involves overcoming several practical and pedagogical challenges. These include difficulties in content creation, technical limitations, and concerns about feasibility (Cevikbas et al., 2023; Zhang, 2023). While technology offers considerable opportunities for enriching mathematical modeling, effective integration depends on strong pedagogical foundations and technological competence. Many PSTs and in-service teachers lack sufficient experience with digital learning environments, resulting in difficulties managing and delivering technology-enhanced instruction. In the absence of proper training and support, these challenges may lead to suboptimal implementation of FL (Lo, 2024).

Additionally, infrastructure limitations present another major barrier to effective FL implementation. Unstable internet access, insufficient digital tools, and inadequate institutional investment in technology can disrupt learning continuity, particularly in regions with lower technological accessibility (Mhlanga et al., 2022; Ndibalema, 2022). The COVID-19 pandemic further underscored these challenges, as educators and students struggled to maintain engagement and access learning materials in remote settings (Izagirre-Olaizola & Morandeira-Arca, 2020).

Institutional support is essential for addressing the challenges associated with FL. Research highlights that successful FL implementation depends on a robust digital infrastructure, comprehensive faculty training, and institutional policies that encourage sustainable technology integration (Polat & Karabatak, 2021). Institutions that invest in learning management systems, high-quality video hosting platforms, and well-structured faculty development programs tend to achieve greater success in adopting FL (Chou et al., 2020; Koh, 2019). Moreover, structured orientation programs for students and ongoing technological support for educators can significantly enhance the effectiveness of FL (Cevikbas & Kaiser, 2022; Yorganci, 2025).

As educational methodologies and digital technologies continue to evolve, FL presents a valuable framework for enhancing mathematical modeling. However, its success hinges on prioritizing institutional investment, educator training, and pedagogically sound technology integration (Yong et al., 2015).

Purpose of the Study

Given both the challenges and opportunities surrounding modeling education, this study aimed to explore how PSTs engage with technology-supported mathematical modeling, particularly within the context of FL. While previous research has examined the role of EVs in modeling education (Cevikbas et al., 2025) and the impact of FL on mathematical thinking (Lee & Shin, 2022; Özen Ünal et al., 2023), limited attention has been paid to how PSTs navigate the integration of FL and mathematical modeling, and how they develop competencies in doing so effectively.

This study focuses specifically on the concept of functions within mathematical modeling, aiming to identify key difficulties and strategies for enhancing the teaching and learning experience. By addressing these aspects, this study contributes to a deeper understanding of how FL and technology integration can improve mathematical modeling and better prepare PSTs for real-world teaching scenarios.

Accordingly, this study sought to answer the following research question: What challenges do PSTs face while engaging in mathematical modeling tasks within a flipped and technology-enhanced learning environment?

Research Methodology

General Background

In this study, a phenomenology design was adopted. Phenomenology is a research design that reveals the essence of individuals' experiences with a particular phenomenon (Creswell, 2020). A phenomenological research design was employed to explore the difficulties faced by PSTs regarding mathematical modeling in a learning environment they encountered for the first time (Van Manen, 1990).

Participants

The study was conducted with fourth-year students enrolled in the mathematics education program at a state university during the spring semester of the 2022–2023 academic year. The participants consisted of 51 PSTs (36 females and 15 males) whose data were determined via the purposive sampling method (Christensen et al., 2015). Accordingly, volunteer PSTs taking the "Modeling in Mathematics Teaching" course were included in the study. To ensure confidentiality, each participant was assigned a pseudonym (e.g., S1, S2, …, S51) in the reporting of findings.

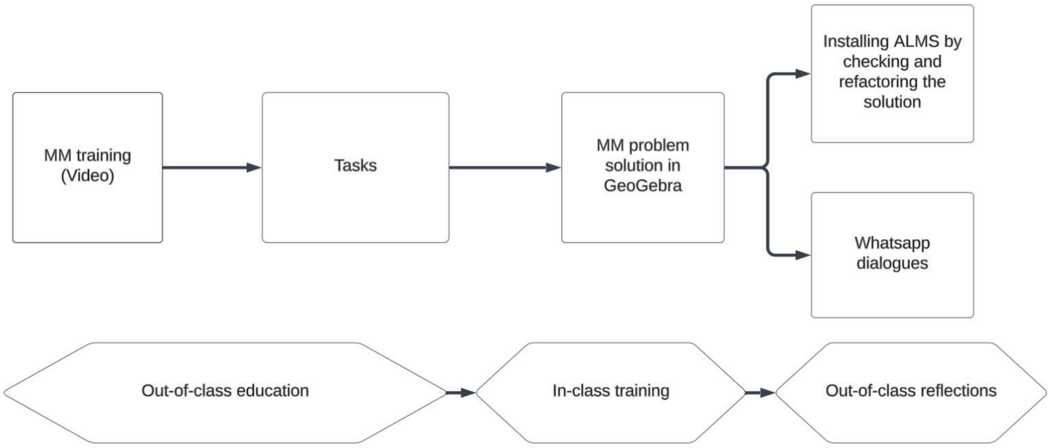
Data Collection

While the training program for PSTs spanned 12 weeks and provided general insights, this study offers a detailed analysis of the second week. Originally planned as a face-to-face course, the training was transitioned to an online format following the earthquake in Türkiye on February 6, 2023.

In the first week, information was provided about the learning process and organization, and the PSTs' duties and responsibilities in the process were explained. The following cycle was followed in each subsequent week: 1. Video lessons, 2. lesson tasks, 3. Geogebra applications, and 4. Studies were conducted using the Academic Learning Management System (ALMS).

The cycle began with out-of-class instructional videos on mathematical modeling, followed by task submissions through the ALMS platform. During synchronous online sessions, PSTs collaboratively solved mathematical modeling problems using GeoGebra, following the modeling process. Afterward, they worked on project tasks asynchronously and uploaded their work to ALMS. In the final stage, participants reflected on their experiences through peer discussions via WhatsApp, allowing for deeper engagement and insight into the challenges encountered (see Figure 1).

Figure 1
Mathematical Modeling Processes in Online and Flipped Classroom Integration



To create the out-of-class video content of the study, the content of the Modeling in Mathematics Teaching course included in the primary school mathematics teaching program of the Board of Higher Education (2018) was used. In this context, mathematical modeling and problem-solving were the focus, and models and modeling processes in mathematics teaching were examined in detail. In addition, the modeling cycle, model development stages, and principles were conveyed to the PSTs. These videos were prepared in a modular structure, lasted 15-20 minutes, and were made accessible on the ALMS platform.

After watching the course videos, the PSTs were responsible for completing homework consisting of three tasks to reinforce the theoretical knowledge they learned and uploaded it to the system within a day.

Modeling problems were solved via GeoGebra software under the supervision of the academician for two hours every week, and the academician provided support to the students via Zoom and the ALMS platform. During this process, the PSTs had the opportunity to experience the mathematical modeling process in practice through authentic problems structured in GeoGebra.

The mathematical modeling problems were regularly solved using GeoGebra and uploaded to the ALMS system. PSTs were given a 36-hour period to re-evaluate the problems, review the data, and repeat the modeling cycle. During this time, they were encouraged to critically reflect on their solutions and make necessary revisions.

The PSTs shared their solution process and the difficulties they experienced on the WhatsApp platform. During this process, the PSTs, who did not have mathematical modeling experience, shared their ideas and perceptions with other PSTs, contributing to modeling learning and developing a deep understanding of the problem. The discussions allowed PSTs to provide feedback on the process and share their experiences.

Data Collection Tools

In this study, data were collected through assignments uploaded by the PSTs to ALMS, GeoGebra recordings and edited versions of modeling problems, WhatsApp dialogs, and semi-structured interviews reflecting the PSTs' views on the process. The study focused only on the Mushroom Problem:

> A cultivated mushroom producer measures the average diameter of the mushrooms as 4 mm on the first day and 20 mm on the fourth day. If this producer should pick the mushrooms when their average diameter is 60 mm, on which day should the producer pick the mushrooms?

The design of the mathematical modeling problem in GeoGebra included the following components: (1) an introductory article, (2) preparatory questions (tasks 1–2), (3) the mushroom problem, and (4) tasks and solution tools reflecting the mathematical modeling process (tasks 3–8). For instance, Task 5 asked, "How would you create a model to solve the problem? Explain." PSTs participated in the process of using at least one of their mobile phones, tablets, or computers. They were able to organize their solutions more effectively and upload them to the ALMS system. A total of 51 records related to the mushroom problem were created during this phase.

Data obtained from semi-structured interviews and WhatsApp correspondence were used in the research. Semi-structured interviews were created to understand the participants' views and experiences on certain topics in depth (Patton, 2015), and the questions were prepared in line with the main themes stated in the literature and the difficulties in the mathematical modeling process. WhatsApp correspondence was preferred to reflect the participants' experiences in their natural environments more authentically (Mann & Stewart, 2000).

Data Analysis

The modeling processes of the PSTs were examined through descriptive analysis. Data sources included their GeoGebra activity records and solution uploads on the ALMS platform. The analysis employed a rubric developed by Hıdıroğlu et al. (2014), grounded in the frameworks of Borromeo Ferri (2006) and Berry and Houston (1995).

The modeling process was evaluated across six core competencies: (1) understanding the problem, (2) selecting variables and making assumptions, (3) mathematization, (4) performing the mathematical solution, (5) interpreting solutions, and (6) verifying the model. Each competency was assessed using a three-level scale: no evidence of an approach, partially appropriate approach, and appropriate approach.

To ensure reliability, all data were systematically examined, and differing interpretations were discussed until consensus was achieved. The rubric criteria are presented in Table 1.

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Table 1
Modeling Competency Rubric (Hıdıroğlu et al., 2014)

Competency	No Evidence	Partially Appropriate	Appropriate
C1: Understanding the Problem	Not understanding the Problem	Shows partial understanding with minor errors.	Demonstrates full understanding by identifying given and required elements.
C2: Choosing Variables and Making Assumptions	Fails to identify relevant variables or make assumptions.	Identifies some variables; assumptions are limited or vague.	Fully identifies variables and assumptions
C3: Mathematization	Lacks or misuses mathematical concepts.	Identifies concepts and partially explains their use.	Accurately identifies and applies mathematical concepts.
C4: Implementing the Mathematical Solution	Provides no solution or incorrect procedure.	Partial solution with errors or no conclusion.	Carries out a complete, accurate mathematical solution.
C5: Interpreting Solutions	Draws no or incorrect conclusions.	Interprets results superficially or with limited real-world relevance.	Draws, interprets, and applies conclusions to real life.
C6: Validating the Model	Provides no validation or incorrect validation.	Attempts partial validation with limited evidence.	Effectively tests and justifies the model in varied situations.

The data obtained from the interview forms were analyzed using the deductive content analysis method (Mayring, 2014), as the categories had been predefined based on the theoretical framework and relevant literature. Content analysis, as described by Miles et al. (2014), involves systematically coding and organizing textual data to identify meaningful patterns. In this study, the analysis process began with the coding of interview and correspondence data according to predetermined categories, followed by the grouping of emerging themes under these categories. Finally, thematic interpretation was conducted by examining the relationships among the categories to construct broader themes that reflect the participants' experiences and challenges within the context of mathematical modeling and flipped learning. Themes were determined via an inductive approach in the content analysis process, the data were grouped according to their similarities and common codes were created. Two researchers independently performed the coding, and then consensus and disagreements were evaluated. Disagreements were resolved through discussion, and the reliability of the evaluators was calculated at 95% confidence intervals (Miles & Huberman, 1994). Codes and themes were presented visually with tables and interpreted in the findings section. In addition, statements from PSTs supporting the findings are also included.

Validity and Reliability

To ensure the validity and reliability of this research, credibility, transferability, consistency, and confirmability strategies were employed (Lincoln & Guba, 1985). To enhance the credibility of the study, the research environment, participant profiles, and emerging themes were described in detail, ensuring that the results accurately reflected the real situation (Shenton, 2004)

To ensure transferability, visuals taken from the written solutions of the PSTs were presented directly, and these visuals supported the PSTs' competencies with concrete examples

(Miles & Huberman, 1994). Consistency was achieved by presenting the findings directly and without interpretation. This approach allows readers to develop interpretations (Patton, 2015). The raw data and coding obtained during the research process were preserved for confirmability. Thus, relevant people can examine the accuracy and transparency of research (Yin, 2018).

Ethical Consideration

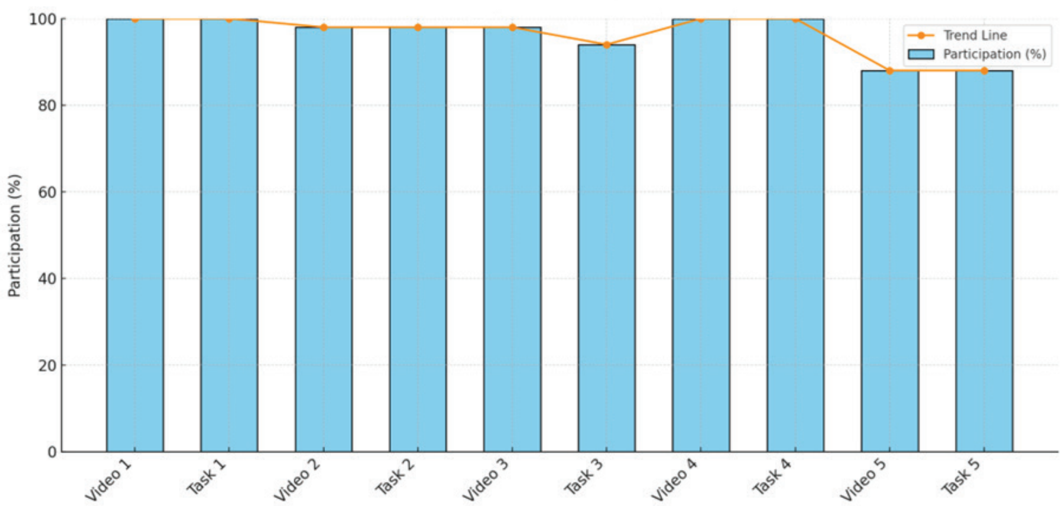
Ethical approval for this study was granted by the Ethics Committee of Van Yuzuncu Yil University (Meeting Date: 23.02.2023, Decision No: 2023/04-13). The instruments used in the research were reviewed and approved for compliance with social and human sciences ethical standards. Participants were informed about the purpose of the study, and informed consent was obtained. Confidentiality and participant well-being were prioritized throughout the study. Furthermore, this study was derived from the master's thesis of the third author, conducted under the supervision of the first author and the co-supervision of the second author.

Research Results

Pre-Classroom Engagement in the FLM

The pre-classroom engagement of PSTs with the video content and tasks uploaded to the ALMS system is presented in Figure 2.

Figure 2
PSTs' Engagement with Video Materials and Tasks



As shown in Figure 2, PSTs demonstrated high levels of participation in pre-classroom activities, suggesting strong engagement with the theoretical foundations of mathematical modeling within the FLM framework. The online FLM supported PSTs' active engagement through flexible access to content and timely feedback. It enabled learning continuity during post-earthquake disruptions and promoted self-regulation and real-world problem-solving via GeoGebra-based tasks. These results suggest that FLM contributed positively to PSTs' conceptual understanding and modeling competencies.

However, the findings also revealed several challenges. Although ALMS ensured access to instructional materials, infrastructure limitations occasionally impeded participation. Inconsistent academic support and varying levels of student engagement in online discussions were also noted. Furthermore, while digital tools enhanced learning, the abundance of content sometimes made it difficult for PSTs to monitor their progress. These findings underscore the importance of a robust digital infrastructure, equitable mentoring systems, and strategies to enhance active participation and the effective use of feedback in technology-supported learning environments.

PSTs' Mathematical Modeling Competencies in the Mushroom Problem

Table 2 presents PSTs' competencies across the stages of the mushroom modeling task.

Table 2
Participants' Competence in the Mushroom Problem

Competence	None (%)	Partially (%)	Suitable (%)
C1	0	0	100
C2	0	0	100
C3	8	84	8
C4	12	80	8
C5	14	78	8
C6	90	8	2

All participants successfully demonstrated competence in understanding the problem (C1) and defining variables/assumptions (C2). However, a sharp decline was observed in mathematization (C3), solution implementation (C4), and interpretation (C5), with the majority showing only partial competency. Notably, model validation (C6) emerged as the most challenging stage, with 90% of participants unable to demonstrate relevant skills.

These results indicate that while PSTs could initially engage with the problem context, they encountered difficulties transitioning into formal mathematical representations and validating their models. The first critical gap was observed at the mathematization phase. Despite diverse modeling attempts, the study emphasized identifying a crystallized model representative of the overall problem-solving process (see Figure 3).

Figure 3
Main Models Used by Participants in the Mushroom Problem

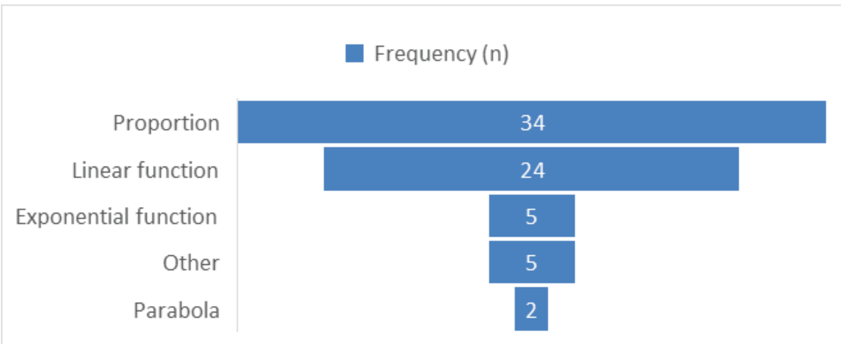
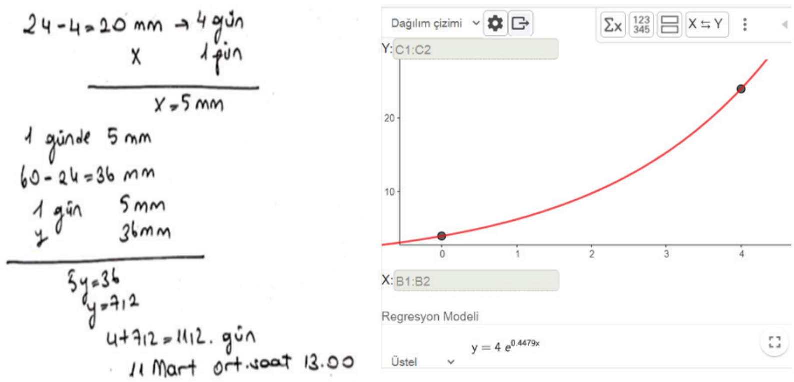


Figure 3 illustrates the mathematical models utilized by PSTs for the mushroom problem. Proportional reasoning, closely associated with slope and the concept of a proportionality constant, emerged as the most frequently adopted model. Within the broader category of function models, the linear function was the dominant choice, whereas only a few participants interpreted the growth pattern as exponential.

Figure 4

Examples of Models in Mathematization Competence

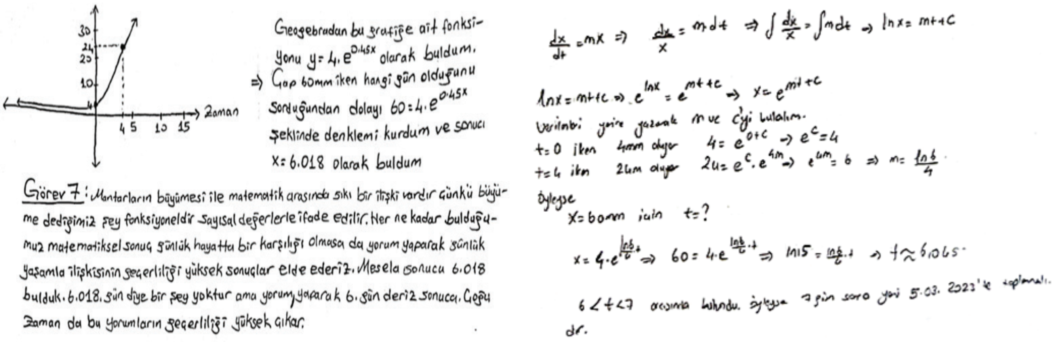


S19's solution S41's solution

Most PSTs, like S19, represented mushroom growth using a proportional model, overlooking the more appropriate exponential nature of the phenomenon. Only four PSTs (S7, S18, S37, S41) demonstrated appropriate modeling within this competency. Notably, S41 correctly employed an exponential model using GeoGebra (rounding the function to) and validated it through a differential equation, exemplifying a high level of modeling competence. However, most participants exhibited only partially appropriate approaches, often implementing flawed models without adequately questioning their validity.

The second critical difficulty emerged in model validation. Many PSTs, despite modeling errors, displayed overconfidence and failed to reflect critically on their solutions. S1 stated, *"I believe the best relationship can be established through a linear function,"* while S6 and S9 expressed confidence in their models based on the given data. This reflects a lack of critical engagement with the modeling process. Although S2 acknowledged data limitations—*"We cannot determine the validity of this relationship in daily life with our limited data"*—no further attempts were made to overcome these constraints. S5 noted: *"In fact, I tried to do it but I could not,"* indicating awareness of alternative models but difficulty in implementation. These findings suggest a need for further development in validation skills, critical thinking, and the integration of technology to enhance model evaluation and refinement.

Figure 5
Actions of S7 and S41 During the Model Validation Process



S7's solution and explanation   S41's alternative solution and model validation

In Figure 5, participant S7 realized that the exponential function could be a correct model by saying, “There is a close relationship between mushroom growth and mathematics because what we call growth is functional… For example, we found the result to be 6.018. We can say it is day 6 by commenting…” However, this attempt could not go beyond interpretation. Participant S41 successfully applied the exponential function model, expressed the model graphically, symbolically, and verbally, and reached the same conclusion with the differential equation:

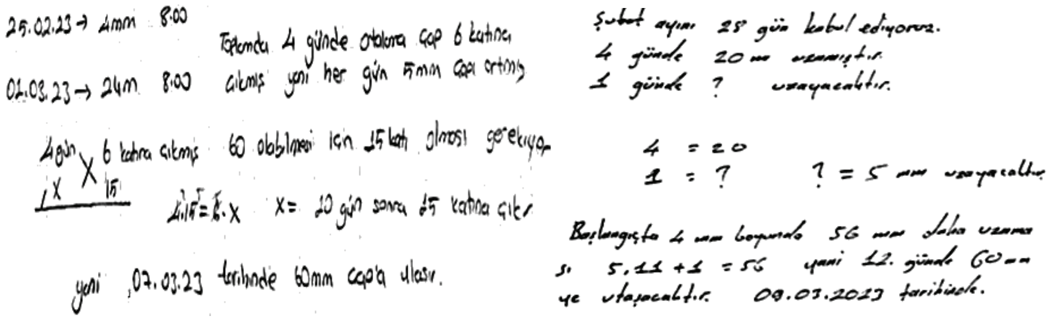
$$\frac{dx}{dt} = mx \Rightarrow \frac{dx}{x} = mdt \Rightarrow \int \frac{dx}{x} = \int mdt \Rightarrow \ln x = mt + c$$
$$\ln x = mt + c \Rightarrow e^{\ln x} = e^{mt+c} \Rightarrow x = e^{mt+c}$$

After determining the constants *m* and *c* by substituting the given values, the PSTs obtained an exponential function representing mushroom growth. In this function, $t \approx 6.045$ corresponds to $x = 60$. Since this value lies between 6 and 7, it was concluded that the mushrooms should be collected on the 7th day. This approach illustrates how mathematical models can yield results compatible with real-life contexts when the modeling process is deeply understood. However, other participants failed to demonstrate appropriate strategies during the model verification phase. Despite the use of technology, the results should be supported by field knowledge or cross-verified with alternative technologies.

Conceptual Errors of PSTs in the Modeling Process

Some PSTs, unlike others, misused mathematical objects in the modeling process of the mushroom problem. These errors mainly concerned the concepts of proportion and function. First, S1’s error regarding the concept of proportion in solving the mushroom problem is given in Figure 6.

Figure 6
Errors Related to the Concept of Proportion for S1 and S16

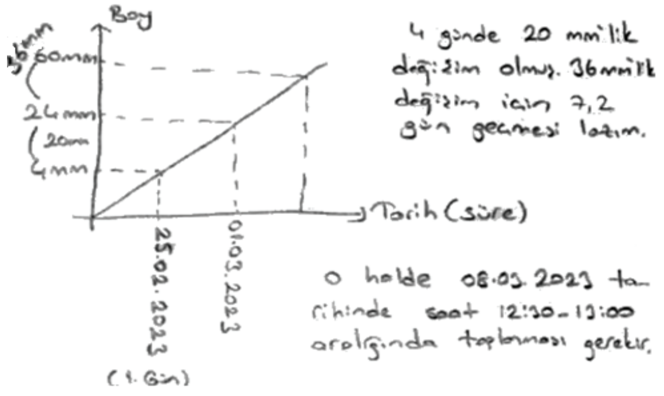


S1 approached the mushroom problem using proportionality and outlined her steps in Figure 6. However, Figure 6 reveals an error in her use of mathematical terminology related to direct proportionality. She incorrectly symbolized the problem as "4=20, 1=?"—a misrepresentation of the unitary ratio between time and the mushroom radius. While she verbally articulated the problem ("*If there is a 20 mm extension in 4 days, how much will it be in 1 day*?") correctly, her symbolic expression was flawed. This suggests that S1 struggled to conceptualize and accurately express proportional relationships mathematically.

Similarly, S16 approached the mushroom problem as the mushroom diameter, initially 4 mm on February 25, 2023, increased by 5 mm daily, reaching 24 mm by March 1. However, S16 mistakenly focused on growth multipliers rather than daily increments, saying, "*The average diameter increased 6 times in 4 days, meaning 5 mm daily; it would need to be 15 times to reach 60.*" As shown in Figure 6, S16 correctly applied proportion but created a new unit by misinterpreting variable relationships. These findings indicate that S16 did not fully internalize the conceptual meaning of proportion, highlighting the importance of PSTs developing both procedural and conceptual understanding in mathematics.

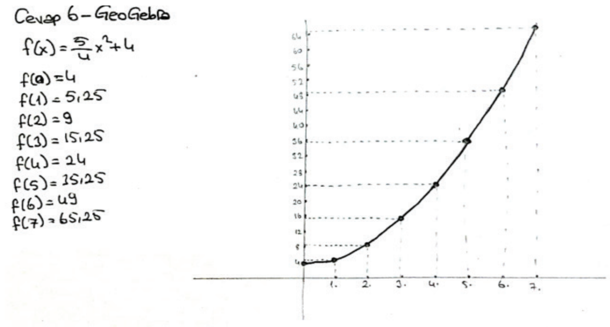
Second, Figure 7 presents the errors made by S21 in constructing the linear function graph for the mushroom problem.

Figure 7
S21's Errors in Assuming that the Linear Function Passes through the Origin



Although the graphs initially appeared correct, they contained significant contextual errors. In the mushroom problem, the diameter increased from 4 mm (25.02.2023) to 24 mm (01.03.2023), and PSTs were asked to estimate when it would reach 60 mm. Many assumed linear growth but misapplied this assumption. A key issue was failing to evaluate whether the graph should pass through the origin. For instance, S21 drew the graph starting at the origin, despite no data prior to 25.02.2023. This led to unitless axes, inaccurate representations, and a lack of technological verification. S21, for example, assumed a 5 mm daily growth but did not validate the intercept, which should have been (0, -1), not the origin.

Figure 8
S5's Quadratic Function Model Error for Mushroom Growth



Finally, S5 used a parabolic model to represent the average radius growth of mushrooms, as shown in Figure 8. The date 25.02.2023 was defined as day 0 in GeoGebra, and the axes were labeled as time (days) and growth (mm). The equation $f(x) = (5/4)x^2 + 4$ was constructed using the points (0,4) and (4,24). However, the exponential nature of mushroom growth—emphasized in many models–was overlooked.

These errors were mainly due to limited conceptual understanding, particularly regarding proportion and function. The findings underscore the importance of both procedural and conceptual knowledge in mathematical modeling.

Reflections from WhatsApp Conversations of PSTs

Primary school mathematics PSTs' WhatsApp discussions provided insights into their experiences and challenges in mathematical modeling.

PSTs displayed varied levels of engagement with mathematical modeling, particularly in distinguishing "mathematical modeling" from "modeling in mathematics". S37 expressed uncertainty: *"Professor Ibrahim [on modeling on YouTube]... clearly separates this in her video, but I was also curious about your opinions."* Similarly, S18 associated mathematical modeling with real-life problems: *"For a situation to be defined as mathematical modeling, there must be a problem that exists in real life."* S26 reflected on encountering real-life applications for the first time, asking, *"Is what we call interpretation meaningful for real life?"* These reflections reveal that while PSTs are familiar with problem-solving, connecting it to real-world applications remains relatively new.

PSTs also expressed confusion about what modeling entails and how to apply theoretical concepts in practice. For example, S37 asked, *"Does using a material or model constitute mathematical modeling, or not? If not, what does mathematical modeling mean?"* S26 admitted that peer discussions only added to her confusion: *"My mind was already confused, friends, and it got even more confused after reading; I would be grateful if you could clarify."*

S36 highlighted the challenge of differing assumptions: *"When you make solutions based on assumptions here, the results also change… everyone has their own truth."* PSTs questioned the necessity of strict theoretical distinctions, as S37 remarked, *"I do not think it makes much sense to define concepts so theoretically."*

These discussions suggest that while PSTs are actively engaging with mathematical modeling, they struggle to distinguish theoretical ideas from practical applications, highlighting the need for more targeted support in linking theory to real-world contexts.

Challenges Related to the Mathematical Modeling Process Based on PSTs' Views

PSTs reported facing challenges in the modeling process regarding their modeling competencies, subject matter knowledge, and use of technology. PSTs' responses concerning whether they experienced difficulties in each modeling competency are presented in Figure 9.

Figure 9

Challenges in the Modeling Competencies of PSTs

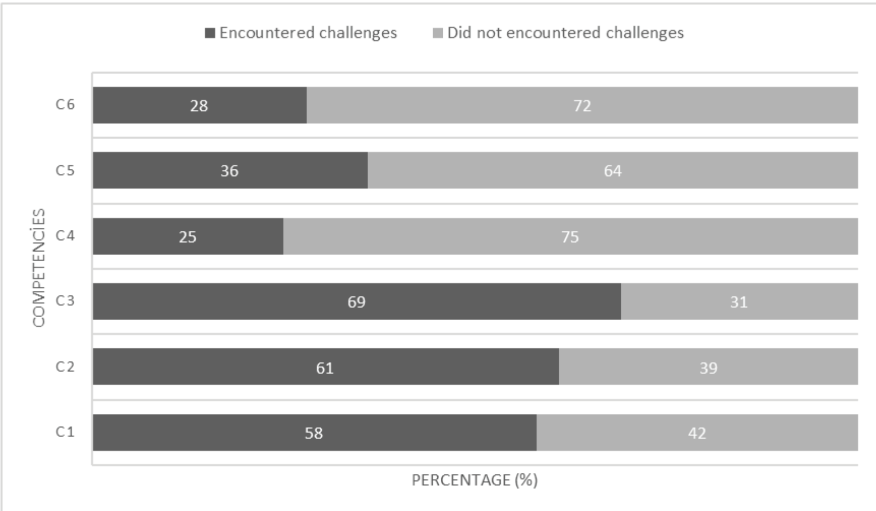


Figure 9 illustrates that PSTs faced challenges at various stages of the mathematical modeling process. Specifically, 58% struggled with understanding the problem, 61% with simplifying it, and 69% with mathematization–the stage where they encountered the most difficulty. In contrast, fewer challenges were reported in mathematical work (25%) and model validation (28%).

Challenges in understanding stemmed largely from unfamiliarity with such problem types. As S23 remarked, *"The reason I struggled was that I hadn't encountered such problems before."* Simplifying the problem was hindered by uncertainty about which data were essential; S17 stated, *"I had trouble because I could not decide whether some data were necessary or not."* The mathematization stage, particularly model creation, proved most demanding: *"Finding the model was the hardest part… it required very creative thinking and good reasoning,"* noted S14.

In the mathematical work stage, concerns arose over the reliability of the model and the risk of calculation errors. S27 shared, *"I was worried about making calculation errors… if my model was not solid, this part became extremely difficult."* Model validation was similarly challenging due to its iterative and time-consuming nature. S16 reflected, *"The validation stage was challenging… revisiting the steps was also difficult. Every time I checked, I felt as if I had*

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made a mistake." Additionally, interpreting results in real-life contexts presented difficulties. As S26 explained, *"Examining the consistency of my results with real life… was the hardest stage."*

These challenges were particularly evident in the initial modeling competencies: understanding the problem, selecting variables, and mathematization. The absence of feedback led PSTs to consider partially correct or incorrect models as accurate. For instance, most PSTs assumed mushroom growth followed a linear or proportional pattern, which influenced subsequent stages of the modeling process. Despite having access to time and resources, they did not adequately verify or validate their solutions. As a result, misconceptions rooted in superficial reasoning remained uncorrected.

This highlights the need for PSTs to develop not only technical modeling skills but also the ability to question and validate their models critically. Errors in early stages negatively impacted later steps, and a lack of evaluative thinking perpetuated inaccurate conclusions. Therefore, providing structured opportunities for validation and reflection is essential for fostering deeper modeling competencies.

PSTs faced notable challenges related to subject matter knowledge, particularly in connecting mathematical concepts, recalling relevant knowledge, and linking mathematics to real-life contexts.

Connecting mathematical concepts was a common difficulty. As S49 noted, *"Even though I knew which mathematical knowledge I needed, I often had a hard time connecting the concepts."* Recalling mathematical knowledge also posed a barrier during problem-solving. S15 stated, *"Sometimes I encountered minor problems due to having forgotten some topics."* Relating mathematics to real life emerged as another significant challenge. S12 admitted, *"I did not know how to translate real-life events into mathematics… I did not have enough knowledge to establish the connection."* Similarly, S37 emphasized the interdependence between problem understanding and concept recall: *"When you do not understand the problem, no mathematical concept comes to mind. When concepts do not come to mind, you cannot form a model or formula."* These findings reveal that gaps in conceptual understanding and application hinder PSTs' ability to effectively engage in modeling tasks.

PSTs encountered challenges related to technology use during the modeling process, primarily due to hardware limitations and insufficient technological proficiency.

Hardware deficiencies impeded modeling efficiency. As S34 stated, *"The difficulty I experienced while solving the problems was related to the inadequacy of my technological equipment,"* highlighting how the lack of essential tools affected their performance. Technological inadequacy was another barrier. S37 noted, *"We received GeoGebra training online, and at that time, I did not have a computer, so I couldn't practice and reinforce the skills,"* indicating that limited access and practice opportunities hindered their ability to use digital tools effectively. These findings suggest that improving both access to technological resources and the development of digital competencies is essential for enhancing the effectiveness of the mathematical modeling process.

Discussion

This study aimed to examine the challenges and learning experiences of PSTs within a technology-supported online learning environment that integrates FL and focuses on mathematical modeling. The findings provide evidence regarding both the potential benefits and the limitations of this instructional model, particularly in fostering problem-solving competencies and enhancing engagement with technology.

The integration of FL and mathematical modeling appears to positively influence PSTs' participation and learning. However, transitioning to a fully online format presents new limitations, including decreased interaction and difficulty in accessing learning materials. While PSTs demonstrated high participation levels, many encountered technical problems–especially during real-time activities involving GeoGebra–which negatively impacted their modeling processes. These results resonate with previous research indicating that technological infrastructure issues and disparities in digital literacy levels among both students and teachers hinder the success of online FL environments (Lepkova et al., 2024).

The study also underscores the need for institutional support. Effective implementation of FL requires not only strong digital infrastructure but also teacher training and well-established institutional policies (Polat & Karabatak, 2021). In line with the literature, studies conducted during the pandemic emphasized the importance of structured institutional guidance and professional development programs for effective adoption of the FLM (Chou et al., 2020).

In terms of modeling competencies, the findings revealed a discrepancy between PSTs' self-assessments and their actual performance. While participants perceived themselves to be successful in the mathematical modeling process, objective data indicated underperformance in critical stages such as model creation and validation. This mismatch may be attributed to limited feedback mechanisms, which hinder PSTs' ability to recognize their errors and contribute to an overestimation of their skills. These observations align with previous studies reporting significant difficulties during the mathematization and validation phases of modeling (Anhalt et al., 2021; Guerrero-Ortiz & Borromeo Ferri, 2022; Şahin et al., 2023) and emphasize the importance of structured support systems in helping PSTs develop modeling competencies (Asempapa & Sturgill, 2019; Hıdıroğlu et al., 2014). Furthermore, Viseu et al. (2020) reported that PSTs struggle with identifying appropriate proportional constants, evaluating the plausibility of model-generated results, and designing graphs based on experimental data. Similarly, our study revealed that PSTs often use linear functions or parabolas instead of exponential functions, suggesting the need for stronger support in the development of accurate mathematical representations.

Another challenge identified was the integration of technology in the modeling process. While some PSTs used digital tools such as GeoGebra effectively for analyzing models, others encountered barriers such as hardware limitations, lack of software proficiency, and technological infrastructure issues. These findings are consistent with prior studies showing that although technology has the potential to enhance mathematical modeling processes, it can also be a source of difficulty for individuals unfamiliar with it (Altuntaş et al., 2024; Hıdıroğlu et al., 2018). Nevertheless, it was observed that PSTs were able to quickly test and evaluate different model types using software tools, even with limited data. This demonstrates that effective use of technology can strengthen modeling skills, particularly when supported by targeted training on tools like MATLAB and GeoGebra (Cevikbas et al., 2022).

The study also revealed that the success of FL depends heavily on institutional backing. Although FL promotes flexibility, active learning, and student responsibility (Johnston, 2017; Lepkova et al., 2024), its effectiveness is often limited by inadequate support structures and digital inequalities (Cevikbas et al., 2025). While student engagement is encouraged, it should go beyond verbal participation to include active involvement in decision-making, peer discussions, and technological engagement in modeling activities. Guerrero et al. (2015) noted that FL enhances student-centered interactions, but its effectiveness over traditional methods depends significantly on how the model is implemented. As Lo (2024) and Johnston (2017) emphasized, student feedback–informed adaptations, such as revising video content or adding supplementary tutorials, can improve engagement. However, in the absence of comprehensive institutional policies and resources, such adaptations may not be sustainable.

A further finding relates to the conceptual understanding of PSTs. Many participants demonstrated difficulties in connecting mathematical concepts or recalling relevant knowledge when needed, especially in the context of real-life problems. This deficiency often led to inaccurate or inappropriate modeling decisions and limited their ability to interpret real-world phenomena mathematically. These challenges are linked to insufficient subject-matter knowledge and a shallow understanding of fundamental mathematical concepts, which are critical for constructing and validating models (Aruna et al., 2022; Erbas et al., 2014; Zulkarnaen, 2018).

Moreover, PSTs often exhibited a tendency to prioritize obtaining final answers over engaging with the modeling process itself. Combined with limited experience and time constraints, this focus hampered the development of robust modeling strategies (Sen Zeytun et al., 2023). Yong et al. (2015) highlighted that interactive learning environments have the potential to strengthen students' conceptual understanding; however, this potential may remain limited in the absence of in-class interaction.

Cognitive errors, such as using irrelevant procedures or having incomplete schemas, highlight fundamental misunderstandings of mathematical concepts (Zulkarnaen, 2018). These errors often stem from deficiencies in algebra, geometry, and trigonometry, which are essential for constructing and interpreting models. Many studies emphasize the need for PSTs to develop a deeper understanding of these areas (Ikeda et al., 2025; Sen Zeytun et al., 2023).

Despite these challenges, the study underscores the potential for PSTs to enhance their modeling abilities through targeted educational interventions. Structured professional development opportunities, workshops, and coursework focused on the practical application of mathematical knowledge can support PSTs in bridging the gap between theoretical understanding and real-world problem-solving (Şahin et al., 2023). Furthermore, leveraging technology and recognizing individual cognitive differences may further support PSTs in developing effective modeling and reasoning skills (Gonzales & Arnal-Bailera, 2020).

Conclusions and Implications

This study demonstrates that a technology-supported online mathematical modeling environment, integrated with the FL model, holds considerable potential for enhancing PSTs' modeling competencies. Nevertheless, its effectiveness is limited by structural challenges such as reduced interaction in online settings, difficulties in accessing learning materials, and technical shortcomings.

The effectiveness of the FL model was found to be closely associated with institutional infrastructure, teacher training, and the availability of digital equipment. Variations in PSTs' levels of technological literacy and disparities in access to digital tools hindered the integration of technology into the modeling processes.

A notable discrepancy emerged between PSTs' self-perceptions and their actual performance in modeling tasks. Specifically, deficiencies were evident during model construction and validation phases. Moreover, PSTs often struggled to connect fundamental mathematical concepts to real-world contexts, showing a tendency to prioritize outcomes over processes. This process-oriented gap hindered the development of effective modeling strategies and limited their ability to interpret real-life situations mathematically.

While integrating FL into mathematical modeling tasks fosters engagement and problem-solving, challenges related to self-regulation and digital fluency must be addressed to fully leverage its potential. It is important to acknowledge that the study's scope—limited to a specific cohort of PSTs—may affect the generalizability of the findings. Furthermore, individual cognitive differences and contextual influences were not directly examined.

Future research should consider longitudinal designs to better capture the progression of PSTs' modeling skills. Comparative investigations exploring diverse instructional designs within FL environments may also offer valuable insights for optimizing teacher education practices.

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Ethical Approval

This study was approved by the Van Yüzüncü Yıl University Ethics Committee for Human Research with protocol number 2023/04-13 and decision number 13/26, dated 23.02.2023.

Declaration of Interest

The authors declare no competing interest.

References

Abeysekera, L., & Dawson, P. (2015). Motivation and cognitive load in the flipped classroom: Definition, rationale, and a call for research. *Higher Education Research & Development*, *34*(1), 1–14. https://doi.org/10.1080/07294360.2014.934336

Al-Khasawneh, A., & Obeidallah, R. (2015). Factors contributing to e-learning success. *International Journal of Information and Communication Technology Education*, *11*(3), 30–38. https://doi.org/10.4018/IJICTE.2015070103

Altuntaş, M., Ay, Z. S., & Çetin, I. (2024). Mathematical modeling in online learning environments: Student challenges. *Bolu Abant İzzet Baysal Üniversitesi Eğitim Fakültesi Dergisi*, *24*(2), 1049–1075. https://doi.org/10.17240/aibuefd.2024..-1407578

Anhalt, C. O., Cortez, R., & Aguirre, J. M. (2021). Mathematical modeling thinking: A construct for developing mathematical modeling proficiency. In J. M. Suh, M. H. Wickstrom, & L. D. English (Eds.), *Exploring mathematical modeling with young learners* (pp. 45–66). Springer. https://doi.org/10.1007/978-3-030-63900-6_3

Aruna, A., Alma, B., Nurbapa, M., Doolotbai, B., & Akmaral, R. (2022). Technology of formation of students' skills of educational and cognitive activity through mathematical modelling. *World Journal on Educational Technology: Current Issues*, *14*(6), 1685–1697. https://orcid.org/0000-0002-5893-6712

Asempapa, R. S. & Sturgill, D. J. (2019). Mathematical modeling: Issues and challenges in mathematics education and teaching. *Journal of Mathematics Research, 11*(5), 71–78. https://doi.org/10.5539/jmr.v11n5p71

Bergmann, J., & Sams, A. (2012). *Flip your classroom: Reach every student in every class every day*. International Society for Technology in Education.

Berry, J., & Houston, K. (1995). *Mathematical modelling*. Gulf Professional Publishing.

Berry, J., & Nyman, M. A. (1998). Introducing mathematical modelling skills to students and the use of posters in assessment. *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, *8*(2), 103-–115. https://doi.org/10.1080/10511979808965888

Blomhøj, M., & Jensen, T. H. (2003). Developing mathematical modelling competence: Conceptual clarification and educational planning. *Teaching Mathematics and its Applications, 22*(3), 123–139. https://doi.org/10.1093/teamat/22.3.123

Blum, W., & Leiβ, D. (2007). How do students and teachers deal with mathematical modelling problems? The example sugarloaf and the DISUM project. In C. Haines, P. L. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 222–231). Horwood.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modeling, applications, and links to other subjects: State, trends and issues in mathematics instruction. *Educational Studies in Mathematics*, *22*(1), 37–68.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM, 38*(2), 86-95.

Borromeo Ferri, R. (2007). *Modelling problems from a cognitive perspective*. In C. Haines, P. Galbraith, W. Blum & S. Khan (Ed.), *Mathematical modelling* (ICTMA-12): Education, engineering and economics (pp. 260–270). Horwood.

Brown, B., Delanoy, N., & Webster, M. (2022, May 18). Flipped learning in grade 7 and 9 mathematics. *Open Technology in Education, Society, and Scholarship Association Conference Proceedings, 2*(1), 1–8. https://doi.org/10.18357/otessac.2022.2.1.23

Bui, T. T., & Nguyen, T. S. (2023). The survey of digital transformation in education: A systematic review. *International Journal of TESOL & Education, 3*(4), 32–51. https://doi.org/10.54855/ijte.23343

Cevikbas, M., & Kaiser, G. (2022). Promoting personalized learning in flipped classrooms: A systematic review study. *Sustainability*, *14*(18), 11393. https://doi.org/10.3390/su141811393

Cevikbas, M., Greefrath, G., & Siller, H. S. (2023). Advantages and challenges of using digital technologies in mathematical modelling education–a descriptive systematic literature review. *Frontiers in Education, 8*, 1142556. https://doi.org/10.3389/feduc.2023.1142556

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2022). A systematic literature review of the current discussion on mathematical modelling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering. *Educational Studies in Mathematics*, *109*(2), 205-236. https://doi.org/10.1007/s10649-021-10104-6

Cevikbas, M., Mießeler, D., & Kaiser, G. (2025). Pre-service mathematics teachers' experiences and insights into the benefits and challenges of using explanatory videos in flipped modelling education. *ZDM–Mathematics Education*, 1–14. https://doi.org/10.1007/s11858-025-01650-x

Chen, Y., Wang, Y., Kinshuk, & Chen, N. S. (2014). Is FLIP enough? Or should we use the FLIPPED model instead? *Computers & Education, 79*, 16–27. https://doi.org/10.1016/j.compedu.2014.07.004

Chou, C. L., Hung, M. L., Tsai, C. W., & Chang, Y. C. (2020). Developing and validating a scale for measuring teachers' readiness for flipped classrooms in junior high schools. *British Journal of Educational Technology*, *51*(4), 1420–1435. https://doi.org/10.1111/bjet.12895

Christensen, L. B., Johnson, R. B., & Turner, L. A. (2015). *Araştırma Yöntemleri Desen ve Analiz* [*Research Methods Design and Analysis*]. (A. Aypay, Ed.). Anı Publishing.

Creswell, J. W. (2020). *Nitel araştırma yöntemleri: Beş yaklaşıma göre nitel araştırma ve araştırma deseni* [Qualitative research methods: Qualitative research and research design according to five approaches]. (M. Bütün & S. B. Demir, Eds.). Siyasal Kitabevi.

Edgar, G. A. (2008). *Measure, topology, and fractal geometry* (2nd ed.). Springer.

Effendi, A. D., Aziz, N. A. A., Low, H. L., & Sia, C. W. (2024). The role of intelligent system in transforming flipped classroom for online learning. In *Intelligent systems modeling and simulation III: Artificial intelligent, machine learning, intelligent functions and cyber security* (pp. 289–332). Springer. https://doi.org/10.1007/978-3-031-67317-7_19

Erbas, A. K., Kertil, M., Çetinkaya, B., Cakiroglu, E., Alacaci, C., & Bas, S. (2014). Mathematical modeling in mathematics education: Basic concepts and approaches. *Educational Sciences: Theory and Practice*, *14*(4), 1621–1627.

Fernández-Martín, F. D., Romero-Rodríguez, J. M., Gómez-García, G., & Navas-Parejo, M. R. (2020). Impact of the flipped classroom method in the mathematical area: A systematic review. *Mathematics*, *8*(12), 2162. https://doi.org/10.3390/math8122162

Frejd, P. (2012). Teachers' conceptions of mathematical modelling at Swedish Upper Secondary school. *Journal of Mathematical Modelling and Application, 1*(5), 17–40.

Fulton, K. (2012). 10 reasons to flip. *Phi Delta Kappan, 94*(2), 20–24. https://doi.org/10.1177/003172171209400205

Galbraith, P., Stillman, G., Brown, J., & Edwards, I. (2007). Facilitating middle secondary modelling competencies. In *Mathematical modelling* (pp. 130–140). Woodhead Publishing.

Gonzales, A., & Arnal-Bailera, A. (2020). Prospective teachers difficulties in integrating technology into problem solving and teaching on rational numbers. *Acta Scientae*, 50–63. https://doi.org/10.17648/acta.scientiae.6237

Greefrath, G., Hertleif, C., & Siller, H. S. (2018). Mathematical modelling with digital tools–A quantitative study on mathematising with dynamic geometry software. *ZDM – Mathematics Education, 50*(1), 233–244. https://doi.org/10.1007/s11858-018-0924-6

Greefrath, G., Siller, H. S., Vorhölter, K., & Kaiser, G. (2022). Mathematical modelling and discrete mathematics: opportunities for modern mathematics teaching. *ZDM–Mathematics Education*, *54*(4), 865-879. https://doi.org/10.1007/s11858-022-01339-5

Guerrero, S., Beal, M., Lamb, C., Sonderegger, D., & Baumgartel, D. (2015). Flipping undergraduate finite mathematics: Findings and implications. *Primus*, *25*(9-10), 814–832. https://doi.org/10.1080/10511970.2015.1046003

Guerrero-Ortiz, C., & Borromeo Ferri, R. (2022). Pre-service teachers' challenges in implementing mathematical modelling: Insights into reality. *PNA. Revista de Investigación en Didáctica de la Matemática, 16*(4), 309–341. https://doi.org/10.30827/pna.v16i4.21329

Haines, C., & Crouch, R. (2007). Mathematical modelling and applications: Ability and competence frameworks. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 417–424). Springer.

Hıdıroğlu, Ç. N. (2015). *Teknoloji destekli ortamda matematiksel modelleme problemlerinin* çözüm *süreçlerinin analizi: Bilişsel ve* üstbilişsel *yapılar* üzerine *bir açıklama* [*Analysis of the solution processes of mathematical modeling problems in technology-supported environment: An explanation on cognitive and metacognitive structures*]. Doktora tezi [PhD thesis]. Dokuz Eylül Üniversitesi, Eğitim Bilimleri Enstitüsü, İzmir [Dokuz Eylul University, Institute of Educational Sciences, Izmir].

Hıdıroğlu, Ç. N., Özaltun, Çelik, A., Kula, Ünver, Z. & Bukova, Güzel, E. (2018). Matematik öğretmeni adaylarının teknoloji destekli matematiksel modelleme sürecindeki eylemleri: Uzaklık problemi [Prospective mathematics teachers' actions in technology-aided mathematical modeling process: distance problem]. *Erzincan Üniversitesi Eğitim Fakültesi Dergisi, 20*(3), 782–809.

Hıdıroğlu, Ç. N., Tekin Dede, A., Kula, S., & Bukova Güzel, E. (2014). Öğrencilerin kuyruklu yıldız problemi'ne ilişkin çözüm yaklaşımlarının matematiksel modelleme süreci çerçevesinde incelenmesi [Examining students' solution approaches to the comet problem within the framework of the mathematical modeling process.]. *Mehmet Akif Ersoy* Üniversitesi *Eğitim Fakültesi Dergisi* [*Mehmet Akif Ersoy University Faculty of Education Journal*]*, 31*, 1–17.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester Jr (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). IAP.

Huesca, G., Martínez-Treviño, Y., Molina-Espinosa, J. M., Sanromán-Calleros, A. R., Martínez-Román, R., Cendejas-Castro, E. A., Bustos, R. (2024). Effectiveness of using ChatGPT as a tool to strengthen benefits of the flipped learning strategy. *Education Sciences, 14*, Article660. https://doi.org/10.3390/educsci14060660

Ikeda, T., Borromeo Ferri, R., & Kawakami, T. (2025). Factors influencing Japanese pre-service teachers' self-efficacy in teaching mathematical modelling. *ZDM–Mathematics Education*, 1–14. https://doi.org/10.1007/s11858-025-01666-3

Izagirre-Olaizola, J., & Morandeira-Arca, J. (2020). Business management teaching-learning processes in times of pandemic: Flipped classroom at a distance. *Sustainability*, *12*(23), 10137. https://doi.org/10.3390/su122310137

Jablonski, S., Barlovits, S., & Ludwig, M. (2023). How digital tools support the validation of outdoor modelling results. *Frontiers in Education, 8*, 15.1145588 https://doi.org/10.3389/feduc.2023.1145588

Johnston, B. M. (2017). Implementing a flipped classroom approach in a university numerical methods mathematics course. *International Journal of Mathematical Education in Science and Technology*, *48*(4), 485–498. https://doi.org/10.1080/0020739X.2016.1259516

Jummai, B. (2021). The need for digital transformation in the education sector in Nigeria. *Current Journal of Applied Science and Technology,* 41–47. https://doi.org/10.9734/cjast/2021/v40i4731636

Jung, I. & Lee, J. (2019). A cross-cultural approach to the adoption of open educational resources in higher education. *British Journal of Educational Technology, 51*(1), 263–280. https://doi.org/10.1111/bjet.12820

Kaiser, G., Schwarz, B., & Buchholtz, N. (2011). Authentic modelling problems in mathematics education. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (Vol. 1, pp. 591–601). Springer. https://doi.org/10.1007/978-94-007-0910-2_57

Khanova, J., Roth, M. T., Rodgers, J. E., & McLaughlin, J. E. (2015). Student experiences across multiple flipped courses in a single curriculum. *Medical education*, *49*(10), 1038–1048. https://doi.org/10.1111/medu.12807

Kiem, M. T., & Keodavan, X. (2024). Unpacking the advantages and challenges of flipped classrooms in initial mathematics teacher education in Vietnam. *Eurasia Journal of Mathematics, Science and Technology Education*, *20*(5), em2437. https://doi.org/10.29333/ejmste/14449

Koh, J. H. L. (2019). Four pedagogical dimensions for understanding flipped classroom practices in higher education: A systematic review. *Educational Sciences: Theory and Practice*, *19*(4), 14–33. https://doi.org/10.12738/estp.2019.4.002

Kozikoğlu, İ. (2019). Analysis of the studies concerning flipped learning model: a comparative meta-synthesis study. *International Journal of Instruction, 12*(1), 851–868. https://doi.org/10.29333/iji.2019.12155a

Leary, M., Tylka, A., Corsi, V., & Bryner, R. (2021). The effect of first-year seminar classroom design on social integration and retention of STEM first-time, full-time college freshmen. *Education Research International, 2021*, 4262905. https://doi.org/10.1155/2021/426290

Lee, J., & Shin, D. (2022). Effects of a flipped classroom using Khan Academy and mathematical modeling on overcoming difficulties in learning mathematics. *Research in Mathematical Education*, *25*(2), 99–115. https://doi.org/10.7468/JKSMED.2022.25.2.99

Lepkova, N., Gülseçen, S., & Talan, T. (2024). Flipped classroom method application case study analysis. *Baltic Journal of Modern Computing, 12*(2), 150–164. https://doi.org/10.22364/bjmc.2024.12.2.02

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Sage Publications.

Lo, C. K. (2024). Strategies for enhancing online flipped learning: A systematic review of empirical studies during the COVID-19 pandemic. *Interactive Learning Environments*, *32*(7), 3517–3545. https://doi.org/10.1080/10494820.2023.2184392

Love, B., Hodge, A., Grandgenett, N., & Swift, A. W. (2014). Student learning and perceptions in a flipped linear algebra course. *International Journal of Mathematical Education in Science and Technology*, 45(3), 317–324. http://dx.doi.org/10.1080/0020739X.2013.822582

Lundin, M., Bergviken Rensfeldt, A., Hillman, T., Lantz-Andersson, A., & Peterson, L. (2018). Higher education dominance and siloed knowledge: A systematic review of flipped classroom research. *International Journal of Educational Technology in Higher Education, 15*(1), 20. https://doi.org/10.1186/s41239-018-0101-6

Mann, C., & Stewart, F. (2000). *Internet communication and qualitative research: A handbook for researching online*. Sage Publications.

Mayring, P. (2014). *Qualitative content analysis theoretical foundation, basic procedures and software solution*. Beltz.

Mhlanga, D., Denhere, V., & Moloi, T. (2022). COVID-19 and the key digital transformation lessons for higher education institutions in South Africa. *Education sciences*, *12*(7), 464. https://doi.org/10.3390/educsci12070464

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook* (2nd ed.). Sage Publications.

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd ed.). Sage Publications.

Ndibalema, P. (2022). Constraints of transition to online distance learning in Higher Education Institutions during COVID-19 in developing countries: A systematic review. *E-learning and Digital Media*, *19*(6), 595–618. https://doi.org/10.1177/20427530221107510

O'Flaherty, J., & Phillips, C. (2015). The use of flipped classrooms in higher education: A scoping review. *The internet and higher education*, *25*, 85–95. https://doi.org/10.1016/j.iheduc.2015.02.002

Okai, S., Uddin, M., Arshad, A., Alsaqour, R., & Shah, A. (2014). Cloud computing adoption model for universities to increase ict proficiency. *Sage Open, 4*(3), Article 215824401454646. https://doi.org/10.1177/2158244014546461

Özen Ünal, D., Yazıcı, E., & Arabacıoglu, T. (2023). Implications from a modelling based flipped mathematics course. *International E-Journal of Educational Studies, 7*(15), 465–483. https://doi.org/10.31458/iejes.1286927

Pattimukay, N., Takaria, J., & Ishabu, L. (2023). The effect of the flipped classroom model on the mathematical critical thinking ability of elementary school students. *Jurnal Pendidikan Matematika (JUPITEK)*, *6*(1), 49–54. https://doi.org/10.30598/jupitekvol6iss1pp49-54

Patton, M. Q. (2015). *Qualitative research & evaluation methods* (4th ed.). Sage Publications.

Polat, H., & Karabatak, S. (2022). Effect of flipped classroom model on academic achievement, academic satisfaction and general belongingness. *Learning Environments Research*, *25*(1), 159–182. https://doi.org/10.1007/s10984-021-09355-0

Priyaadharshini, M., & Sundaram, B. V. (2018). Evaluation of higher-order thinking skills using learning style in an undergraduate engineering in flipped classroom. *Computer Applications in Engineering Education, 26*(6), 2237–2254. https://doi.org/10.1002/cae.22035

Sen Zeytun, A., Cetinkaya, B., & Erbas, A. K. (2023). Why do prospective teachers have difficulties in mathematical modelling? Insights from their perspectives. *International Journal of Mathematical Education in Science and Technology*, 1–24. https://doi.org/10.1080/0020739X.2023.2171922

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information*, 22(2), 63–75.

Siller, H. S., & Greefrath, G. (2010). Mathematical modelling in class regarding to technology. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.), *Proceedings of the sixth congress of the European Society for Research in Mathematics Education* (pp. 2136–2145). INRP.

Silva, R., Martins, F., Costa, C., Cravino, J., & Lopes, J. B. (2021). Learning Scenario to Promote Comprehension of the Meaning of Subtraction. *Education Sciences*, *11*(12), 757. https://doi.org/10.3390/educsci11120757

Strayer, J. F. (2012). How learning in an inverted classroom influences cooperation, innovation and task orientation. *Learning Environments Research, 15*(2), 171–193. https://doi.org/10.1007/s10984-012-9108-4

Sulaimon, J. T., & Manditereza, B. (2024). Investigating the effect of the traditional flipped classroom in teaching primary 3 class mathematics. *Journal of Education and Practice*, *8*(1), 1–12. https://orcid.org/0000-0003-2564-5860

Şahin, S., Gürbüz., R. & Dogan, M. F. (2023). Investigating mathematical modeling problem designing process of inservice mathematics teachers. *Cukurova University Faculty of Education Journal, 52*(1), 33–70. https://doi.org/10.14812/cuefd.1033080

Truong, T., & Diep, Q. (2023). Technological spotlights of digital transformation in tertiary education. *IEEE Access, 11*, 40954–40966. https://doi.org/10.1109/access.2023.3270340

Van Manen, M. (1990). *Researching lived experience: Human science for an action sensitive pedagogy*. The University of Western Ontario.

Viju, G. K. (2021). Online and distance learning In Sudanese Universities: A necessity in the light of the Covid-19 pandemic. *International Journal of Engineering Applied Sciences and Technology*, *5*(10), 11–19.

Viseu, F., Martins, P.M., & Leite, L. (2020). Prospective Primary School Teachers' Activities when Dealing with Mathematics Modelling Tasks. *Journal on Mathematics Education, 11*(2), 301–318. http://doi.org/10.22342/jme.11.2.7946.301-318

Wilkins, J. L. (2010). Modeling quantitative literacy. *Educational and Psychological Measurement*, *70*(2), 267–290. https://doi.org/10. 1177/0013164409344506

Yang, C. C. R., & Chen, Y. (2020). Implementing the flipped classroom approach in primary English classrooms in China. *Education and Information Technologies*, *25*(2), 1217–1235. https://doi.org/10.1007/s10639-019-10012-6

Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). Sage

Yong, D., Levy, R., & Lape, N. (2015). Why no difference? A controlled flipped classroom study for an introductory differential equations course. *Primus*, *25*(9-10), 907–921. https://doi.org/10.1080/10511970.2015.1031307

Yoon, S., Kim, S., & Kang, M. (2020). Predictive power of grit, professor support for autonomy and learning engagement on perceived achievement within the context of a flipped classroom. *Active Learning in Higher Education*, *21*(3), 233-247. https://doi.org/10.1177/1469787418762463

Yorganci, S. (2025). The impact of synchronous online discussions and online flipped learning on student engagement and self-regulation among preliminary undergraduates in a basic math course. *Educational technology research and development*, 1–32. https://doi.org/10.1007/s11423-025-10459-0

Yükseköğretim Kurulu, [Board of Higher Education] (2018). *İlköğretim matematik öğretmenliği lisans programı* [Elementary mathematics teaching undergraduate program]. Board of Higher Education.

Zhang, H. (2023, June). Hierarchical Teaching Practice Based on Majors and Applications of Mathematica in Advanced Mathematics. In *2023 4th International Conference on Education, Knowledge and Information Management (ICEKIM 2023)* (pp. 33–41). Atlantis Press.

Zulkarnaen, R. (2018, October). *Why is mathematical modeling so difficult for students?* In *AIP Conference Proceedings* (Vol. 2021, No. 1, Article 060003). AIP Publishing. https://doi.org/10.1063/1.5062790

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