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The future muon collider for the research of the anomalous neutral quartic $Z\gamma\gamma\gamma$, $ZZ\gamma\gamma$, and $ZZZ\gamma$ couplings

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Abstract In the post-LHC era, the muon collider represents a frontier project capable of providing high-energy and high-luminosity leptonic collisions among future lepton–lepton particle accelerators. In addition, it provides significantly cleaner final states than those produced in hadron collisions. With this expectation in mind, in this article, we research the sensitivity of the anomalous neutral gauge boson couplings $Z\gamma\gamma\gamma$, $ZZ\gamma\gamma$, and $ZZZ\gamma$ defined by dimension-8 operators, through the $\mu^+\mu^- \to \mu^+\mu^- Z\gamma$ signal, with the Z-boson decaying to a neutrino pair. The projections at the future muon collider with a center-of-mass energy of $\sqrt{s} = 10$ TeV, integrated luminosity of $\mathcal{L} = 10$ ab^{-1}, and systematic uncertainties of $\delta_{sys} = 0\%, 3\%, 5\%$, give an expected sensitivity on the anomalous $f_{T,j}/\Lambda^4$ couplings at 95% confidence level of the order of $\mathcal{O}(10^{-3} - 10^{-1})$ TeV^{-4}. Compared with the research of the ATLAS and CMS Collaborations on the anomalous quartic gauge couplings, we find that the high-luminosity future muon collider could have better sensitivity.

1 Introduction

In recent years, the interest of the high-energy scientific community in the design and construction of a possible future muon collider (MuCol) with multi-TeV energies has been resumed and intensified [1–3]. In this regard, among the post-Large Hadron Collider (LHC) generation of particle accelerators, the MuCol represents a frontier project with the capability to provide very high-energy leptonic collisions and open the path to physics programs beyond the Standard Model (BSM).

The MuCol offers the possibility of collision of particles to very high energies with relatively small circular colliders. Different stages of energies of 3, 6, 10, 14 TeV, etc., and very high luminosity are being proposed. In addition, due to their clean collider environment, this type of collider provides a great tool to search for new physics in the electroweak sector, especially by producing multiple electroweak vector bosons. The Standard Model (SM) Lagrangian contains interaction terms involving three or four electroweak gauge bosons (γ, Z, W). These correspond to the Triple Gauge Couplings (TGC) and Quartic Gauge Couplings (QGC). Studying TGC and QGC is essential to test the SM and find evidence of new physics. The SM predicts QGC that contain at least two charged W bosons ($W^+W^-W^+W^-$, W^+W^-ZZ, $W^+W^-\gamma\gamma$, $W^+W^-Z\gamma$), while the neutral gauge boson couplings ($ZZZZ$, $ZZZ\gamma$, $ZZ\gamma\gamma$, $Z\gamma\gamma\gamma$, $\gamma\gamma\gamma\gamma$) are excluded and they are considered to be effects of physics BSM [4].

One popular approach to studying the anomalous quartic gauge couplings (AQGC) is the concept of Effective Field Theory (EFT), where to parameterize BSM physics, one can extend the SM Lagrangian by adding higher dimension operators, as is explained in Section II, as well as in Refs. [5–10]. In this paper, we study the $\mu^+\mu^- \to \mu^+\mu^- Z\gamma \to \mu^-\mu^+\nu\bar{\nu}\gamma$ scattering process at a future muon collider sensitive to AQGC. The relevant effective interactions are encoded by dimension-8 operators that lead to the Feynman diagrams shown in Fig. 1.

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The most stringent experimental limits arise from vector-boson-scattering processes at the LHC, for which the ATLAS and CMS [11–15] have determined limits of [−0.12, 0.11], [−0.12, 0.11], [−0.28, 0.28], [−0.50, 0.50], [−0.40, 0.40], [−0.90, 0.90], [−0.43, 0.43], [−0.92, 0.92] TeV^{-4}, for the operators $f_{T,j}/\Lambda^4$ with $j = 0, 1, 2, 5, 6, 7, 8, 9$. Limits have also been set by these collaborations using the $pp \to Z\gamma\gamma \to l^+l^-\gamma\gamma$ process [16–18], and by the CMS and TOTEM Collaborations using the $pp \to pZZp$ process [19, 20]. Previously reported limits include those by the L3 and OPAL Collaborations at the Large Electron Positron collider [21, 22], and by the D0 and CDF Collaborations at the Tevatron at Fermilab [23, 24]. In summary, AQGCs have been studied at previous, present, and proposed hadron–hadron, hadron–lepton, and lepton–lepton colliders [4, 25–50].

In the particular case of the anomalous quartic neutral interactions some studied processes are $e^+e^- \to Z\gamma\gamma$ [51, 52], $e^+e^- \to ZZ\gamma$ [53], $e^+e^- \to qq\gamma\gamma$ [54], $e^-\gamma \to ZZe^-$ [55], $e^-\gamma \to Z\gamma e^-$ [56], $\gamma\gamma \to ZZ$ [57], $\gamma\gamma \to \gamma Z$ [58], $\gamma\gamma \to \gamma\gamma$ [59], $e^+e^- \to ZZ\gamma$ [60], $pp \to Z\gamma jj$ [61], $pp \to p\gamma\gamma p \to pZ\gamma p$ [62], $p\bar{p} \to Z\gamma\gamma$ [63], $pp(\bar{p}) \to \gamma\gamma ll$ [64], $pp \to p\gamma^*\gamma^*p \to pZZp$ [65–67], $pp \to p\gamma^*p \to pZZqX$ [68], $pp \to p\gamma^*p \to p\gamma ZqX$ [69], $pp \to qq\gamma ll$ [70], and $pp \to Z\gamma\gamma$ [71]. In the case of the future MuCol the reader can consult Refs. [72–78].

In the following, after introducing our theoretical framework for the AQGC (Sect. 2), we present our strategy for testing for AQGC at the future MuCol in Sect. 3, and finally, we offer our conclusion in Sect. 4.

2 Effective field theory for non-standard interactions of gauge bosons

In this section, we briefly collect the various standard and non-standard interactions implied by the effective Lagrangian (1), as in our previous papers [5, 6, 8–10, 59], as well as in other papers where the formalism of the AQGC has been widely discussed in the literature [55, 64, 70, 79–82]. The EFT extends the SM by adding the higher-dimensional operators that are gauge invariant under the SM:

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_{k=0}^{1} \frac{f_{S,k}}{\Lambda^4} O_{S,k} + \sum_{i=0}^{7} \frac{f_{M,i}}{\Lambda^4} O_{M,i} + \sum_{j=0,1,2,5,6,7,8,9} \frac{f_{T,j}}{\Lambda^4} O_{T,j}, \tag{1}$$

where each $O_{S,k}$, $O_{M,i}$, and $O_{T,j}$ is a gauge-invariant operator of dimension-8, while $\frac{f_{S,k}}{\Lambda^4}$, $\frac{f_{M,i}}{\Lambda^4}$, and $\frac{f_{T,j}}{\Lambda^4}$ are the anomalous effective coefficients. A virtue of the EFT formulation is that one can obtain reliable bounds on the effective coefficients $\frac{f_{S,k}}{\Lambda^4}$, $\frac{f_{M,i}}{\Lambda^4}$, and $\frac{f_{T,j}}{\Lambda^4}$. These bounds are obtained from general considerations. According to the operator structure, there are three classes of genuine AQGC operators, as shown in Eq. (3) in Ref. [80].

Having defined our effective Lagrangian in Eq. (1), we can more precisely state the dimension-8 operators that contain contributions of operators with covariant derivatives, contributions of operators with gauge field strength tensors, and contributions of operators with both covariant derivatives and field strength tensors, as follows:

- Operators with the Higgs boson field ($\frac{f_{S,k}}{\Lambda^4} O_{S,k}$):

$$O_{S,0} = [(D_\mu \Phi)^\dagger (D_\nu \Phi)] \times [(D^\mu \Phi)^\dagger (D^\nu \Phi)], \tag{2}$$

$$O_{S,1} = [(D_\mu \Phi)^\dagger (D^\mu \Phi)] \times [(D_\nu \Phi)^\dagger (D^\nu \Phi)], \tag{3}$$

$$O_{S,2} = [(D_\mu \Phi)^\dagger (D_\nu \Phi)] \times [(D^\nu \Phi)^\dagger (D^\mu \Phi)]. \tag{4}$$

- Operators with Higgs and gauge boson fields ($\frac{f_{M,i}}{\Lambda^4} O_{M,i}$):

$$O_{M,0} = Tr[W_{\mu\nu} W^{\mu\nu}] \times [(D_\beta \Phi)^\dagger (D^\beta \Phi)], \tag{5}$$

$$O_{M,1} = Tr[W_{\mu\nu} W^{\nu\beta}] \times [(D_\beta \Phi)^\dagger (D^\mu \Phi)], \tag{6}$$

$$O_{M,2} = [B_{\mu\nu} B^{\mu\nu}] \times [(D_\beta \Phi)^\dagger (D^\beta \Phi)], \tag{7}$$

$$O_{M,3} = [B_{\mu\nu} B^{\nu\beta}] \times [(D_\beta \Phi)^\dagger (D^\mu \Phi)], \tag{8}$$

$$O_{M,4} = [(D_\mu \Phi)^\dagger W_{\beta\nu} (D^\mu \Phi)] \times B^{\beta\nu}, \tag{9}$$

$$O_{M,5} = [(D_\mu \Phi)^\dagger W_{\beta\nu} (D^\nu \Phi)] \times B^{\beta\mu} + h.c., \tag{10}$$

$$O_{M,7} = [(D_\mu \Phi)^\dagger W_{\beta\nu} W^{\beta\mu} (D^\nu \Phi)]. \tag{11}$$

- Operators with gauge boson fields ($\frac{f_{T,j}}{\Lambda^4} O_{T,j}$):

$$O_{T,0} = Tr[W_{\mu\nu} W^{\mu\nu}] \times Tr[W_{\alpha\beta} W^{\alpha\beta}], \tag{12}$$

Table 1 The QGC modified by the listed dimension-8 operators are shown with ✓

Operators	$WWWW$	$WWZZ$	$ZZZZ$	$WW\gamma Z$	$WW\gamma\gamma$	$ZZZ\gamma$	$ZZ\gamma\gamma$	$Z\gamma\gamma\gamma$	$\gamma\gamma\gamma\gamma$
O_{S0}, O_{S1}, O_{S2}	✓	✓	✓						
O_{M0}, O_{M1}, O_{M7}	✓	✓	✓	✓	✓	✓	✓		
$O_{M2}, O_{M3}, O_{M4}, O_{M5}$		✓	✓	✓	✓	✓	✓		
O_{T0}, O_{T1}, O_{T2}	✓	✓	✓	✓	✓	✓	✓	✓	✓
O_{T5}, O_{T6}, O_{T7}		✓	✓	✓	✓	✓	✓	✓	✓
O_{T8}, O_{T9}			✓			✓	✓	✓	✓

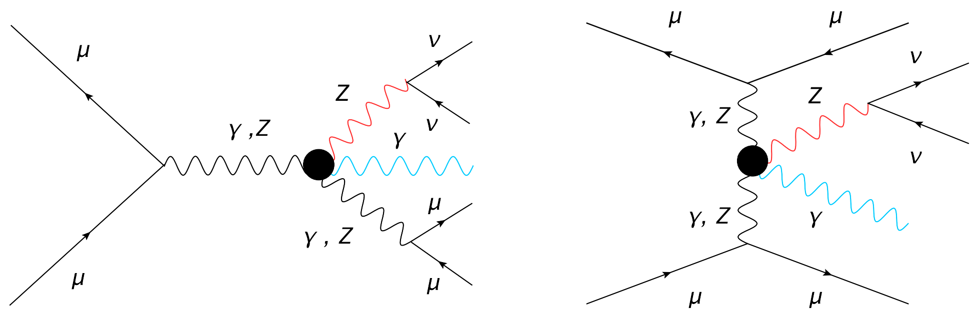

Fig. 1 Feynman diagrams of the process $\mu^-\mu^+ \to \mu^-\mu^+ Z\gamma$ including the anomalous contribution of $ZZ\gamma\gamma$, $Z\gamma\gamma\gamma$, and $ZZZ\gamma$ vertices. New physics (represented by a black circle) in the electroweak sector can induce the neutral AQGC

$$O_{T,1} = Tr[W_{\alpha\nu}W^{\mu\beta}] \times Tr[W_{\mu\beta}W^{\alpha\nu}], \tag{13}$$

$$O_{T,2} = Tr[W_{\alpha\mu}W^{\mu\beta}] \times Tr[W_{\beta\nu}W^{\nu\alpha}], \tag{14}$$

$$O_{T,5} = Tr[W_{\mu\nu}W^{\mu\nu}] \times B_{\alpha\beta}B^{\alpha\beta}, \tag{15}$$

$$O_{T,6} = Tr[W_{\alpha\nu}W^{\mu\beta}] \times B_{\mu\beta}B^{\alpha\nu}, \tag{16}$$

$$O_{T,7} = Tr[W_{\alpha\mu}W^{\mu\beta}] \times B_{\beta\nu}B^{\nu\alpha}, \tag{17}$$

$$O_{T,8} = B_{\mu\nu}B^{\mu\nu}B_{\alpha\beta}B^{\alpha\beta}, \tag{18}$$

$$O_{T,9} = B_{\alpha\mu}B^{\mu\beta}B_{\beta\nu}B^{\nu\alpha}. \tag{19}$$

It is worth mentioning that the subscripts S, T, and M in the operators given by Eqs. (2)–(19) refer to scalar (or longitudinal), transverse, and mixed. These correspond to covariant derivatives of the Higgs field for the longitudinal part and field strength tensors for the transverse part. In the set of genuine AQGC operators given in Eqs. (2)–(19), Φ stands for the Higgs doublet, the covariant derivatives of the Higgs field are given by $D_\mu\Phi = (\partial_\mu + igW^j_\mu\frac{\sigma^j}{2} + \frac{i}{2}g'B_\mu)\Phi$, where $\sigma^j (j = 1, 2, 3)$ represent the Pauli matrices, and $W^{\mu\nu}$ and $B^{\mu\nu}$ are the gauge field strength tensors for the gauge groups $SU(2)_L$ and $U(1)_Y$.

In this paper, we study the operators that lead to AQGC without an ATGC counterpart. The AQGC interactions caused by each operator are shown in Table 1. The $\mu^+\mu^- \to \mu^+\mu^- Z\gamma$ signal is especially sensitive to the anomalous coefficients $f_{T,j}/\Lambda^4$, $j = 0$, 1, 2, 5, 6, 7, 8, 9. For this reason, we consider only these operators in our study. The sensitivities obtained for the $f_{T,8}/\Lambda^4$ and $f_{T,9}/\Lambda^4$ couplings are of particular interest because they can be extracted only by studying the production of electroweak neutral bosons. The contributions of the $O_{T,0}$ and $O_{T,1}$ operators to the examined process are the same [61], so we only explicitly study one of the corresponding couplings.

3 $\mu^+\mu^- Z\gamma$ production and sensitivities on anomalous couplings

3.1 Number of events and total cross-section

We focus on the possibility of direct measurement of the total cross-section of the associated production channel $\mu^+\mu^- \to \mu^+\mu^- Z\gamma$, with anomalous couplings $f_{T,j}/\Lambda^4$, $j = 0-2, 5-9$ at a MuCol, and in the framework of the EFT. The Feynman diagrams with the $Z\gamma\gamma\gamma$, $ZZ\gamma\gamma$, and $ZZZ\gamma$ vertices are shown in Fig. 1

The total cross-section of the reaction $\mu^+\mu^- \to \mu^+\mu^- Z\gamma \to \mu^-\mu^+\nu\bar{\nu}\gamma$ contains the following contributions:

$$\sigma_{Tot}\left(\sqrt{s}, \frac{f_{T,j}}{\Lambda^4}\right) = \sigma_{SM}(\sqrt{s}) + \sigma_{(Int)}\left(\sqrt{s}, \frac{f_{T,j}}{\Lambda^4}\right) + \sigma_{NP}\left(\sqrt{s}, \frac{f^2_{T,j}}{\Lambda^8}\right), \quad j = 0-2, 5-9, \tag{20}$$

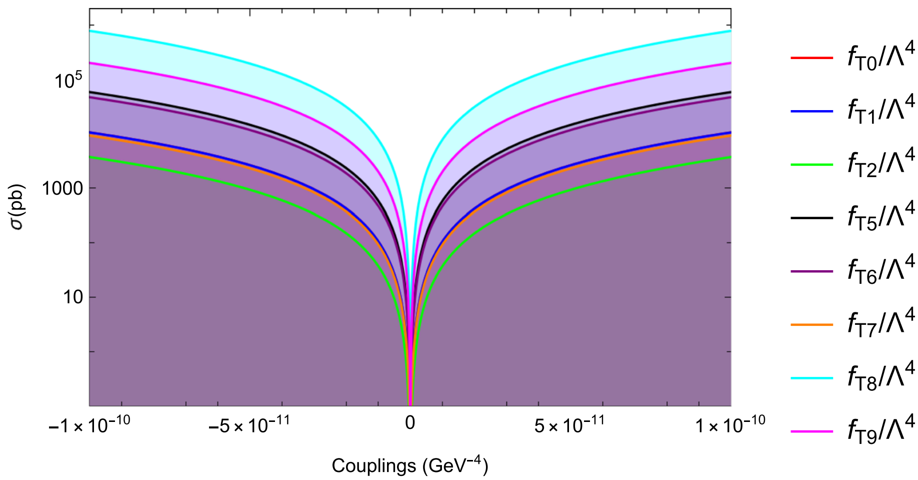

Fig. 2 Total cross-section for the process $\mu^+\mu^- \to \mu^+\mu^- Z\gamma$ with the Z-boson decaying to a neutrino pair, and in terms of the anomalous $f_{T,0}/\Lambda^4$, $f_{T,1}/\Lambda^4$, $f_{T,2}/\Lambda^4$, $f_{T,5}/\Lambda^4$, $f_{T,6}/\Lambda^4$, $f_{T,7}/\Lambda^4$, $f_{T,8}/\Lambda^4$, and $f_{T,9}/\Lambda^4$ couplings for the muon collider with $\sqrt{s} = 10$ TeV. This figure is for purely illustrative purposes. For a perturbative coupling $f < 1$ the EFT is valid for effective coupling values ??$< 10^{-16}$ GeV^{-4}

Table 2 Particle-level selection cuts for the $\mu^+\mu^- Z\gamma$ signal at the muon collider

Kinematical cuts	Anomalous couplings $f_{T,j}/\Lambda^4$ for j = 0, 1, 2, 5, 6, 7, 8 and 9
Cut-1	$\|\eta^\gamma\| < 2.5$
Cut-2	$\not{E}_T > 1000$ GeV GeV
Cut-3	$p_T^\gamma > 800$ GeV

where σ_{SM} is the SM, σ_{Int} is the interference term between the SM and the EFT operators, and σ_{NP} is the contribution of new physics.

In Fig. 2, the process $\mu^+\mu^- \to \mu^+\mu^- Z\gamma$ with the Z-boson decaying to a neutrino pair at $\sqrt{s} = 10$ TeV is analyzed by individually varying the parameters $f_{T,j}/\Lambda^4$, while keeping the remaining coefficients fixed at zero to isolate their contributions. When all coefficients are zero, the cross-section aligns with the SM prediction (σ_{SM}). This methodology facilitates the computation of total cross-sections for varying coupling, revealing deviations from the SM. The findings illustrate the muon collider's capability to probe these anomalous couplings, establishing its effectiveness in exploring aQGCs within the EW sector.

The numerical calculation has been implemented with the help of the MadGraph5_aMC@NLO [83] code, where the operators given in Eqs. (12)–(19) are implemented into MadGraph5_aMC@NLO through the FeynRules package [84] as a Universal FeynRules Output (UFO) module [85]. We have used the requirements on the rapidity, missing transverse energy, and transverse momentum of the outgoing photons, $|\eta^\gamma| < 2.5$ (Cut-1), $\not{E}_T > 1000$ GeV (Cut-2), $p_T^\gamma > 800$ GeV (Cut-3) given in Table 2 to suppress the background process $\mu^-\mu^+ \to \mu^-\mu^+\nu\bar{\nu}\gamma$.

We have used the kinematical distributions of the final state particles with generator-level cuts (minimal cuts applied by MadGraph) to obtain the optimized cuts and reveal the separation region. The distributions are composed of five sets of the anomalous couplings $f_{T,0,2,5,7,8}/\Lambda^4$+SM as signal and the non-resonant process $\mu^-\mu^+ \to \mu^-\mu^+\nu\bar{\nu}\gamma$ as background. Here, the SM refers to the cross-section of the process $\mu^+\mu^- \to \mu^+\mu^- Z\gamma \to \mu^-\mu^+\nu\bar{\nu}\gamma$ fixing all EFT couplings to zero.

Figure 3 illustrates the behavior of the number of events of the $\mu^+\mu^- \to \mu^+\mu^- Z\gamma \to \mu^-\mu^+\nu\bar{\nu}\gamma$ signal and of the background process, as a function of the photon transverse momentum p_T^γ. From the figure, it can be seen that for $p_T^\gamma > 800$ GeV, the majority of the signal remains while the majority of background is removed.

Figure 4 shows the photon $\not{E}_T$ distributions for the predicted signal from $\mu^+\mu^- Z\gamma$ AQGC for $f_{T,0,2,5,7,8}/\Lambda^4$ and relevant background at the MuCol. The figure shows that the majority of signal has $\not{E}_T > 1000$ while the majority of background has $\not{E}_T$ below this value.

3.2 Sensitivity of the anomalous $Z\gamma\gamma\gamma$, $ZZ\gamma\gamma$, and $ZZZ\gamma$ vertices at the future muon collider

In this subsection, we evaluate the sensitivity limits at 95% C.L. of the eight Wilson coefficients $f_{T,j}/\Lambda^4$, $j = 0, 1, 2, 5, 6, 7, 8, 9$ through the $\mu^+\mu^- \to \mu^+\mu^- Z\gamma \to \mu^-\mu^+\nu\bar{\nu}\gamma$ signal, and for the energy and luminosity of a possible future muon collider with $\sqrt{s} = 10$ TeV, $\mathcal{L} = 10$ ab^{-1}. A simple and practical method to estimate the sensitivity of the $f_{T,j}/\Lambda^4$ parameters is based on the chi-square distribution:

$$\chi^2(f_{T,j}/\Lambda^4) = \left(\frac{\sigma_{BSM}(\sqrt{s}, f_{T,j}/\Lambda^4) - \sigma_{SM}(\sqrt{s}) - \sigma_{BG}(\sqrt{s})}{[\sigma_{SM}(\sqrt{s}) + \sigma_{BG}(\sqrt{s})]\sqrt{(\delta_{st})^2 + (\delta_{sys})^2}}\right)^2, \tag{21}$$

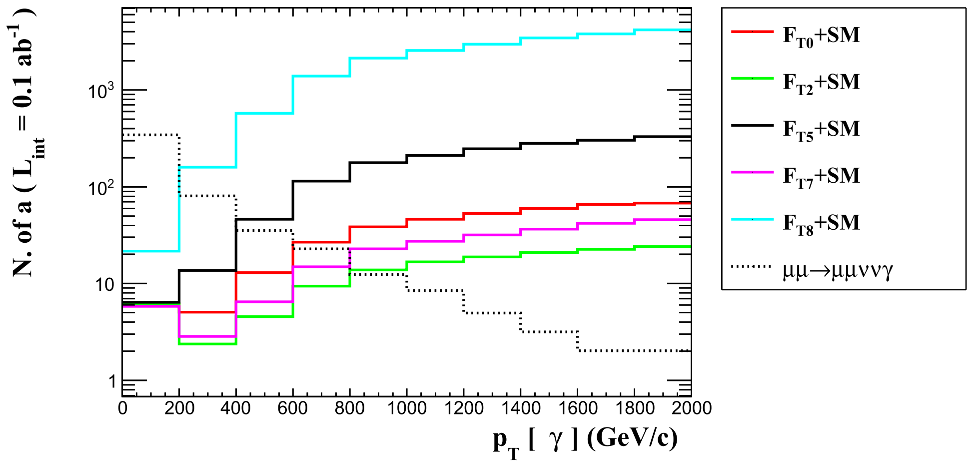

Fig. 3 The number of expected events as a function of photon transverse momentum p_T^γ for the $\mu^-\mu^+ \to \mu^-\mu^+ Z\gamma \to \mu^-\mu^+(\nu\bar{\nu})\gamma$ signal and backgrounds at the MuCol with $\sqrt{s} = 10$ TeV. The distributions are for $f_{T,0,2,5,7,8}/\Lambda^4$ and relevant background. In this figure, we have taken a value of 0.1 TeV^{-4} for each anomalous coupling. This value is chosen for illustrative purposes and is well above that which would correspond to a perturbative coupling $f = 1$ and an energy scale larger than the center-of-mass energy of the collider (10 TeV)

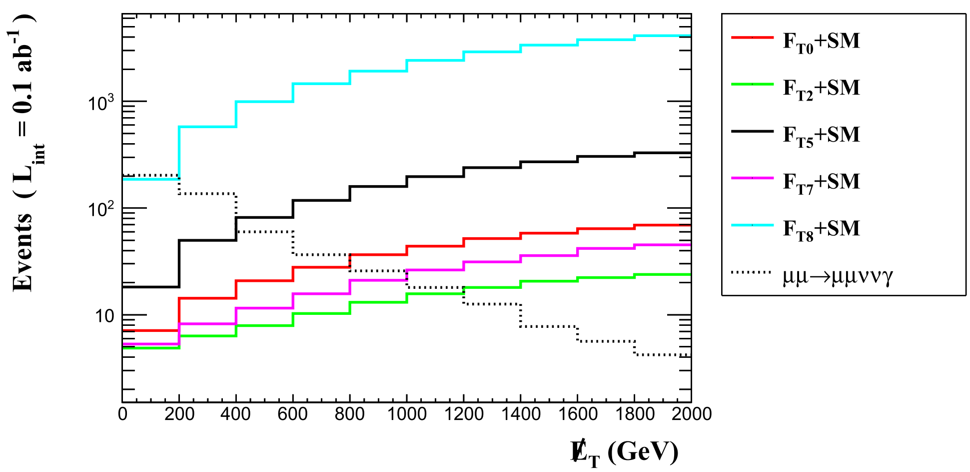

Fig. 4 The number of expected events as a function of the missing transverse energy $\not{E}_T$ (GeV) for the $\mu^-\mu^+ \to \mu^-\mu^+ Z\gamma \to \mu^-\mu^+(\nu\bar{\nu})\gamma$ signal with $f_{T,0,2,5,7,8}/\Lambda^4$ and relevant backgrounds at the MuCol. In this figure, we have taken a value of 0.1 TeV^{-4} for each anomalous coupling. This value is chosen for illustrative purposes and is well above that which would correspond to a perturbative coupling $f = 1$ and an energy scale larger than the center-of-mass energy of the collider (10 TeV)

where $\sigma_{SM}(\sqrt{s})$ is $\mu^+\mu^- \to \mu^+\mu^- Z\gamma$ and $\sigma_{BG}(\sqrt{s})$ is the non-resonant $\mu^+\mu^- \to \mu^-\mu^+\nu\bar{\nu}\gamma$ process, respectively. $\sigma_{BSM}(\sqrt{s}, f_{T,j}/\Lambda^4)$ represents the cross-section in the presence of BSM interactions, $\delta_{st} = \frac{1}{\sqrt{N_{SM}}}$ is the statistical error and δ_{sys} is the relative systematic uncertainty. The primary sources of systematic uncertainty stem from the cross-section predictions of both signal and background processes. Other factors contributing to uncertainty include integrated luminosity, efficiency in identifying photons, and uncertainties associated with the energy-momentum scales and resolutions of final-state particles. While a detailed discussion of systematic uncertainty sources is not the primary focus of this study, our goal is to explore the overall impact of systematic uncertainty on the sensitivities of AQGC. Therefore, we examine three scenarios of systematic uncertainty to assess its effects comprehensively. We choose $\delta_{sys} = 0\%$, 3%, and 5% for our numerical analysis. The number of events is given by $N_{SM} = \mathcal{L} \times \sigma_{SM}$, where $\mathcal{L}$ is the integrated luminosity of the MuCol.

Table 3 lists the Wilson coefficients $f_{T,j}/\Lambda^4$, $j = 0, 1, 2, 5, 6, 7, 8, 9$ and their dependence on the assumed systematic uncertainty. From this table, the anomalous couplings $f_{T,5}/\Lambda^4$, $f_{T,6}/\Lambda^4$, $f_{T,8}/\Lambda^4$, and $f_{T,9}/\Lambda^4$ show a greater sensitivity, of the order of $\mathcal{O}(10^{-3} - 10^{-2})$ TeV^{-4}, compared to the others remaining in the set. In addition, our results show up to 2–3 orders of magnitude better sensitivity compared to the experimental results reported by the ATLAS and CMS Collaborations at the LHC [11–18], as well as many other experimental and phenomenological results. The obtained results on AQGC in the study are competitive with the results reported in papers [72–78].

Finally, Fig. 5 illustrates the comparison of the most sensitive experimental limits and projected sensitivity on the anomalous $f_{T,0}/\Lambda^4$, $f_{T,1}/\Lambda^4$, $f_{T,2}/\Lambda^4$, $f_{T,5}/\Lambda^4$, $f_{T,6}/\Lambda^4$, $f_{T,7}/\Lambda^4$, $f_{T,8}/\Lambda^4$, and $f_{T,9}/\Lambda^4$ couplings for the center-of-mass energy and integrated luminosity expected of $\sqrt{s} = 10$ TeV and $\mathcal{L} = 10$ ab^{-1}, at the future MuCol. The figure shows a significant difference between the experimental results [11–15] and the projected sensitivities. These results indicate that our study improves the existing analyses on the AQGC of the proposed present and future hadron–hadron, hadron–lepton, and lepton–lepton colliders, and provides further guidance for the viable processes in the search for new physics through the anomalous neutral $Z\gamma\gamma\gamma$, $ZZ\gamma\gamma$, and $ZZZ\gamma$ couplings that can be investigated in a possible future muon collider at the frontiers of energy and luminosity.

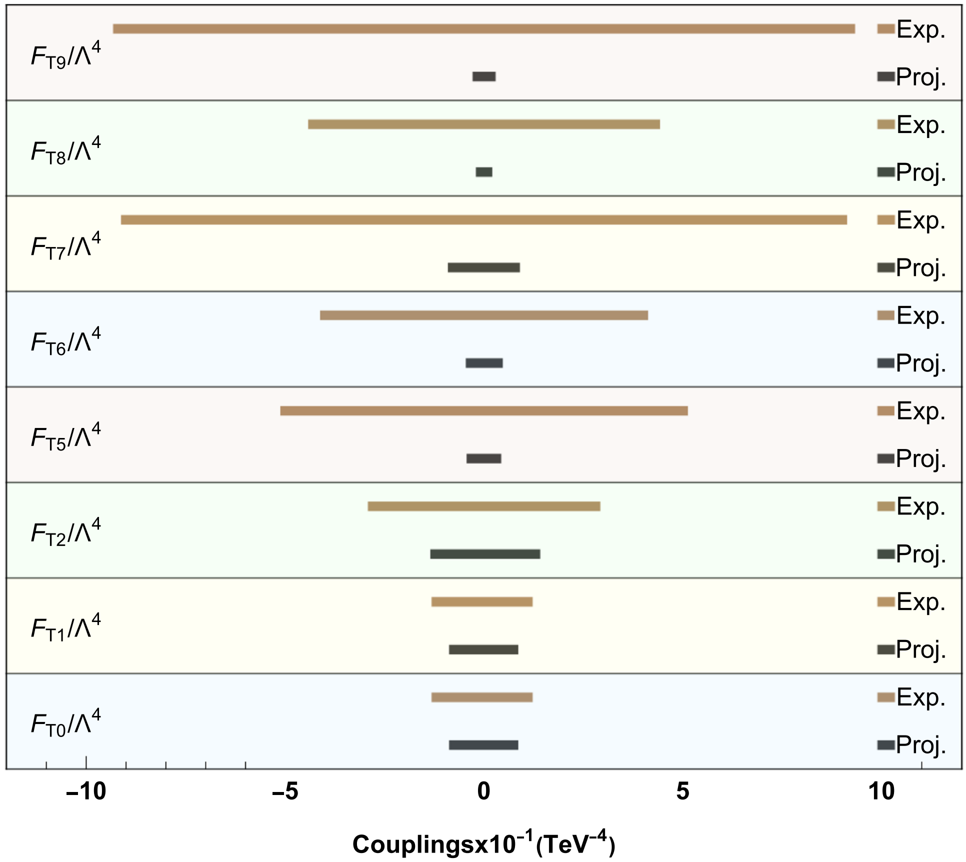

Fig. 5 Comparison of the current experimental limits [11–15] and projected sensitivity on the anomalous $f_{T,0}/\Lambda^4$, $f_{T,1}/\Lambda^4$, $f_{T,2}/\Lambda^4$, $f_{T,5}/\Lambda^4$, $f_{T,6}/\Lambda^4$, $f_{T,7}/\Lambda^4$, $f_{T,8}/\Lambda^4$, and $f_{T,9}/\Lambda^4$ couplings for expected luminosity of $\mathcal{L} = 10\ \text{ab}^{-1}$ with $\sqrt{s} = 10$ TeV at the muon collider. Here, the x-axis illustrates the Couplings $\times 10^{-1}$. For a perturbative value of $f(< 1)$, the valid EFT region is $< 10^{-4}$ TeV^{-4} for a 10 TeV muon collider

4 Conclusion

In this paper, we have evaluated the cross-section for the $\mu^+\mu^- \to \mu^+\mu^- Z\gamma \to \mu^-\mu^+\nu\bar{\nu}\gamma$ process in $\mu\mu$ collisions at a center-of-mass energy of 10 TeV corresponding to an integrated luminosity of 10 ab^{-1}. In addition, we choose three values for the systematic uncertainty of $\delta_{sys} = 0\%,\ 3\%,$ and 5%.

The total cross-sections are calculated in a fiducial region where simulated signal events are selected at generator level in the $\mu^+\mu^- Z\gamma$ channel by requiring exactly two muons and at least one photon with $p_T^{\gamma} > 800$ GeV. The photon candidate must have pseudorapidity $|\eta^{\gamma}| < 2.5$, and events must satisfy $\not{E}_T > 1000$ GeV.

With the elements mentioned above, we estimate projected sensitivities on the anomalous quartic gauge couplings f_{Tj}/Λ^4 using the $\mu^+\mu^- \to \mu^+\mu^- Z\gamma$ channel at the center-of-mass energy of $\sqrt{s} = 10$ TeV and integrated luminosity of $\mathcal{L} = 10$ ab^{-1} for a possible future MuCol. The results reported in this paper are given in Fig. 5 and Table 3, respectively.

In this article, we have shown the importance of the future high-energy and high-luminosity muon collider for the search of deviations from the SM via the anomalous $Z\gamma\gamma\gamma$, $ZZ\gamma\gamma$, and $ZZZ\gamma$ vertices encoded in the Wilson coefficients f_{Tj}/Λ^4. For collider energy equal to 10 TeV and $\delta = 0\%$, a sensitivity to the Wilson coefficients of $f_{T8}/\Lambda^4 = [-8.56, 8.57] \times 10^{-3}$ TeV^{-4}, $f_{T9}/\Lambda^4 = [-1.71, 1.69] \times 10^{-2}$ TeV^{-4}, and $f_{T5}/\Lambda^4 = [-3.24, 3.07] \times 10^{-2}$ TeV^{-4} can be reached by the future MuCol. This is significantly better than the latest results of the ATLAS and CMS Collaborations, where the sensitivity for $f_{T,j}/\Lambda^4$, with $j = 0$, 1, 2, 5, 6, 7, 8, 9 is $\mathcal{O}(10^{-1})$ [11–15]. A comparison between the MuCol predictions and the measurements made by the ATLAS and CMS Collaborations for the anomalous $Z\gamma\gamma\gamma$, $ZZ\gamma\gamma$, and $ZZZ\gamma$ couplings shows that the future MuCol could achieve a sensitivity of up to one order of magnitude better than latest results of ATLAS and CMS. In this sense, the paper could be relevant for the scientific community to prioritize future searches and experimental efforts.

Without a doubt, the future MuCol represents a unique project in elementary particle physics that can provide very high-energy and high-luminosity muon collisions and open the path to a vast and primarily unexplored physics program.

Table 3 Expected sensitivities at 95% C.L. to the anomalous $ZZ\gamma\gamma$, $Z\gamma\gamma\gamma$, and $ZZZ\gamma$ couplings through the process $\mu^+\mu^- \to \mu^+\mu^- Z\gamma \to \mu^-\mu^+\nu\bar{\nu}\gamma$ at the muon collider, for systematic uncertainties of $\delta_{sys} = 0\%, 3\%, 5\%$, center-of-mass energy of 10 TeV and the integrated luminosity of $\mathcal{L} = 10$ ab^{-1}

Couplings (TeV^{-4})	Experimental results (ATLAS and CMS)	Systematic errors	Our projection
		$\delta = 0\%$	$[-7.62, 7.36] \times 10^{-2}$
f_{T0}/Λ^4	$[-1.20, 1.10] \times 10^{-1}$ [11]	$\delta = 3\%$	$[-7.81, 7.55] \times 10^{-2}$
		$\delta = 5\%$	$[-7.95, 7.69] \times 10^{-2}$
		$\delta = 0\%$	$[-7.62, 7.36] \times 10^{-2}$
f_{T1}/Λ^4	$[-1.20, 1.10] \times 10^{-1}$ [11]	$\delta = 3\%$	$[-7.81, 7.55] \times 10^{-2}$
		$\delta = 5\%$	$[-7.95, 7.69] \times 10^{-2}$
		$\delta = 0\%$	$[-1.23, 1.29] \times 10^{-1}$
f_{T2}/Λ^4	$[-2.80, 2.80] \times 10^{-1}$ [11]	$\delta = 3\%$	$[-1.27, 1.32] \times 10^{-1}$
		$\delta = 5\%$	$[-1.29, 1.34] \times 10^{-1}$
		$\delta = 0\%$	$[-3.24, 3.07] \times 10^{-2}$
f_{T5}/Λ^4	$[-5.00, 5.00] \times 10^{-1}$ [12]	$\delta = 3\%$	$[-3.32, 3.15] \times 10^{-2}$
		$\delta = 5\%$	$[-3.37, 3.21] \times 10^{-2}$
		$\delta = 0\%$	$[-3.43, 3.53] \times 10^{-2}$
f_{T6}/Λ^4	$[-4.00, 4.00] \times 10^{-1}$ [12]	$\delta = 3\%$	$[-3.52, 3.62] \times 10^{-2}$
		$\delta = 5\%$	$[-3.58, 3.69] \times 10^{-2}$
		$\delta = 0\%$	$[-7.88, 7.78] \times 10^{-2}$
f_{T7}/Λ^4	$[-9.00, 9.00] \times 10^{-1}$ [12]	$\delta = 3\%$	$[-8.08, 7.98] \times 10^{-2}$
		$\delta = 5\%$	$[-8.22, 8.13] \times 10^{-2}$
		$\delta = 0\%$	$[-8.56, 8.57] \times 10^{-3}$
f_{T8}/Λ^4	$[-4.30, 4.30] \times 10^{-1}$ [13]	$\delta = 3\%$	$[-8.77, 8.78] \times 10^{-3}$
		$\delta = 5\%$	$[-8.93, 8.94] \times 10^{-3}$
		$\delta = 0\%$	$[-1.71, 1.69] \times 10^{-2}$
f_{T9}/Λ^4	$[-9.20, 9.20] \times 10^{-1}$ [13]	$\delta = 3\%$	$[-1.75, 1.73] \times 10^{-2}$
		$\delta = 5\%$	$[-1.78, 1.76] \times 10^{-2}$

For a perturbative value of $f(< 1)$, the valid EFT region is $< 10^{-4}$ TeV^{-4} for a 10 TeV muon collider

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Data Availability All data generated or analyzed during this study are included in this article.

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